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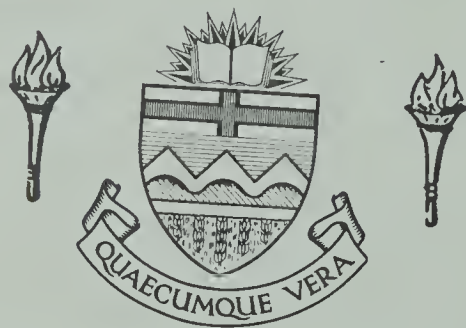
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THE UNIVERSITY OF ALBERTA
MAGNETO-TELLURIC PROFILING
OVER A DEEP STRUCTURE

by



WAYNE JACOBSON PEEPLES

A THESIS
SUBMITTED THE FACULTY OF GRADUATE STUDIES
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE
OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF PHYSICS

EDMONTON, ALBERTA

FALL, 1969

UNIVERSITY OF ALBERTA
FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read,
and recommend to the Faculty of Graduate Studies for
acceptance, a thesis entitled MAGNETO - TELLURIC
PROFILING OVER A DEEP STRUCTURE submitted by Wayne
Jacobson Peeples in partial fulfilment of the require-
ments for the degree of Doctor of Philosophy.

ABSTRACT

Measurements of the surface electric and magnetic fields in the period range 1 - 1,000 seconds were made along two profiles in central and southern Alberta, Canada to determine the crustal resistivities of this region. The analog data obtained was electronically converted to digital form and computer programs were written to analyse large amounts of data using the Fast Fourier transform technique. Calculations were made of the power spectra, the magnetotelluric impedance tensor and the polarization characteristics of geomagnetic micropulsations. The magnetotelluric impedance tensor elements were computationally rotated into the principal directions of resistivity and converted to apparent resistivities which exhibit a strong resistivity anisotropy. This anisotropy is tentatively identified with inhomogeneities in the basement composition and/or the lateral inhomogeneity represented by the Rocky Mountain system. The magnetotelluric results were compared with other geophysical results and indicate the existence of deep lying structure slightly to the south of a fault associated with a graben investigated by Clowes, Kanasewich and Cumming (1968).

ACKNOWLEDGEMENTS

I wish to sincerely thank Dr. D. Rankin for suggesting and supervising this research problem and for his continuous encouragement and support throughout this project.

I am also grateful to Mr. M. D. Burke who designed and built the recording and digitizing equipment and to Mr. N. Ouellette and others who spent many long hours collecting and digitizing the data.

Mr. I. K. Reddy gave much assistance in writing the computer programs and his collaboration on this project has been invaluable.

I would like to thank Mr. G. Sande who supplied the basic Fast Fourier Transform programs and whose knowledge of this technique saved many long hours of work.

My sincere thanks to the numerous other people who have helped to further this research and who have forwarded my academic studies.

Mrs. Mary Thorn is to be commended for her excellent job of typing this thesis.

Throughout the course of his doctoral studies the writer was financially supported by a Graduate Teaching Assistantship from the Department of Physics, University of Alberta and by grants from the National Research Council of Canada.

Finally I would like to acknowledge the contribution of my wife, Joanne, for her extreme patience, work, and understanding during the last months of deadline pressure.

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CHAPTER I

INTRODUCTION

1.1 Purpose of the Investigation

Measurements of the electric and orthogonal magnetic fields at the surface of the earth were made at a series of stations forming the profile shown in Figure 1.3b. These measurements were made during the summer months of 1966 through 1968 as part of a program to determine the electrical properties of the earth's crust under the sedimentary basin of western Canada.

The original purpose of this research was to develop the technology of analysis in order to use the magnetotelluric method as a tool for the study of the crust and upper mantle which together with other geophysical methods could provide a more coherent picture of the subsurface structure. The recent seismic studies of Clowes, Kanasewich and Cumming (1968) have revealed a target for the application of the method. Kanasewich (1968) has proposed a large scale deeply buried structure trending north-east from the Kimberley region of British Columbia into Saskatchewan. The study of this structure by magnetotelluric methods will be the geological goal of this work.

1.2 A Historical Review of the Magnetotelluric Method

Although Clement as early as 1859 noted the coincidence

of telluric currents in North American and European telegraph lines with auroral activity, G. B. Airy (1868) was the first to make a systematic study of the telluric and magnetic field relationships. Terada (1917), Hirayama (1934) and Hatakeyama (1938) measured the dependence of the magnetic and telluric field relationships on the conductivity of the ground.

In 1950, Tikhonov first suggested using natural electromagnetic disturbances for crustal "sounding", and Cagnaird in 1953 published a comprehensive paper on the theory and graphical interpretation of the magnetic and telluric field relationships to determine subsurface electrical resistivities. By assuming the disturbance fields were plane electromagnetic waves, harmonically varying in time, which impinged on a horizontally stratified medium, Cagnaird presented a simple technique for matching theoretical apparent resistivity curves derived from layered models with measured apparent resistivities. Investigations by Wait (1954) and Price (1962) showed the effect of finite source dimensions on the measured apparent resistivities. However, Madden and Nelson (1964) showed that for reasonable earth conductivity models, the plane wave assumption appeared to be valid in the frequency range of interest to the magnetotelluric method.

Neves (1957) extended the theory to consider more complicated models with particular emphasis on two dimensional geometries using the method of finite differences. Several other workers have considered special two dimensional models,

the vertical contact fault within a layer over a uniform basal half space was solved analytically by d'Erceville and Kunetz (1962) and this solution was extended by Rankin (1962) to the case of the vertical dike problem. Cantwell (1960), and Bostick and Smith (1962) proposed techniques to obtain the directions of the principal axes of a conductivity structure for an anisotropic earth while O'Brien and Morrison (1967) solved analytically the general n-layer anisotropic boundary value problem. Madden and Nelson (1964), Sims and Bostick (1969), Morrison et al (1968) and others have indicated analytic techniques to calculate the elements of the magnetotelluric impedance matrix. This matrix relates the telluric and geomagnetic fields for two and three dimensional structures.

Magnetotelluric investigations from a variety of locations in North America, Europe, and Russia have been reported by Vozoff, Hasegawa, and Ellis (1963), Srivastava, Douglas and Ward (1963), Caner and Auld (1968), Whitham and Anderson (1966), Vozoff et al (1969), Berdichevsky (1966), and many others. A compilation of known apparent resistivity curves was assembled by Fournier (1963).

Some interpretation of magnetotelluric results from various arrays and profiles has been attempted. Swift (1967) studied an electrical conductivity anomaly in the south-west United States, and in 1968 Vozoff and Swift reported on studies of a salt basin in northern Germany.

1.3 The Geology and Structure of Southern Alberta

The region involved in this investigation lies in the sedimentary basin of western Canada. A cross section of sedimentary features in this area is shown in Figure 1.3a. This cross section begins at Nordegg in the Alberta foothills and passes through Leduc (see Figure 1.3b) making the cross section approximately perpendicular to the strike of the Rocky Mountain System. Since the vertical scale of Figure 1.3a has been expanded by a factor of 42 it is seen that the sedimentary layers are locally horizontal and uniform in thickness. This basin was thought to be horizontally isotropic electrically and, therefore, an ideal location for short period magnetotelluric analysis.

Burwash (1957) determined from wells that the depth to the Precambrian surface ranges from about 1.43 kilometers near the Saskatchewan border to about 3.6 kilometers at the base of the foothills.

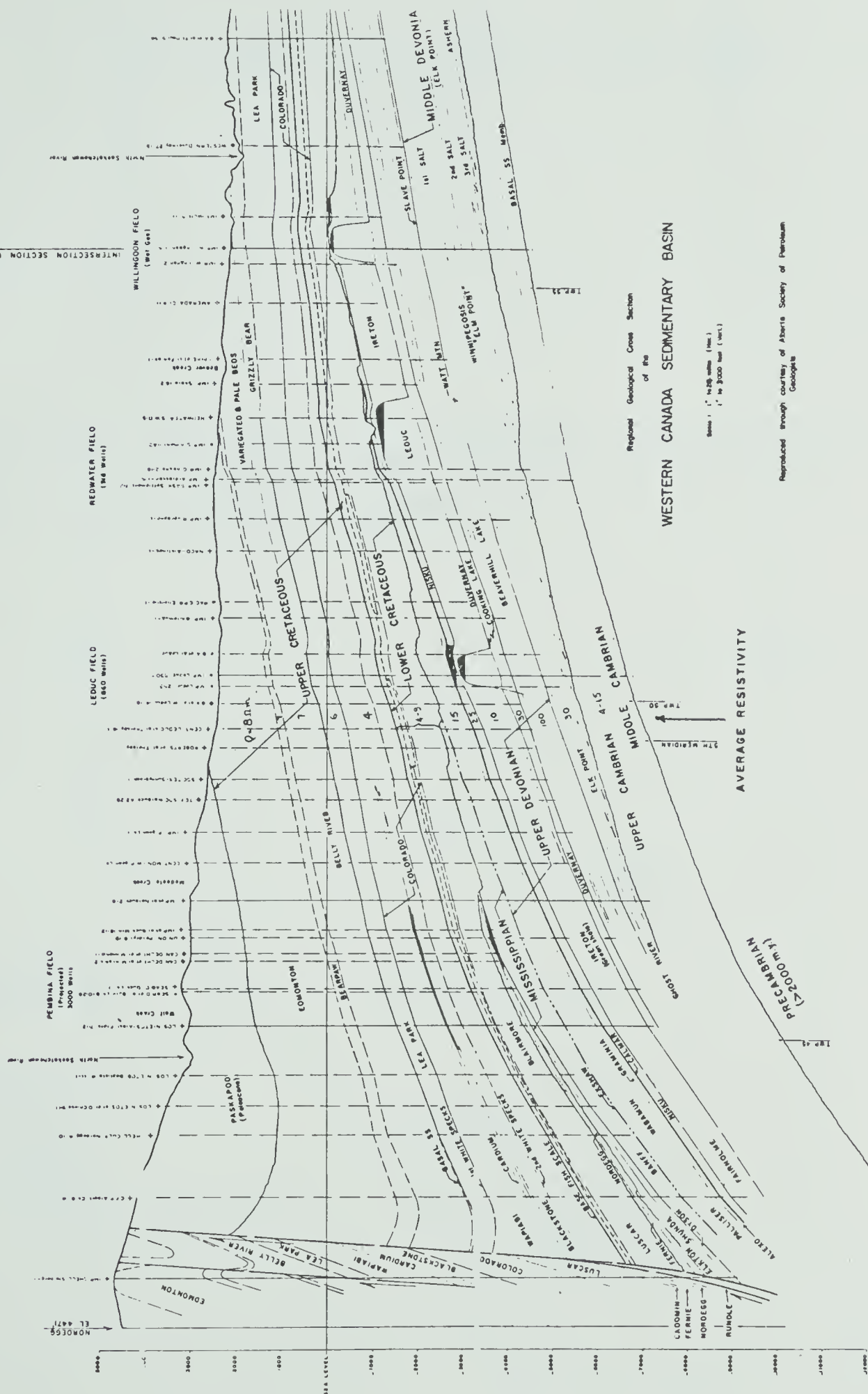
Garland and Burwash (1959) have shown that the major part of the Bouguer gravity anomaly field over central Alberta must be attributed to lithological changes in the Precambrian basement. The lithological changes could lead to an anisotropy in electrical conductivity due to material anisotropy and/or other lateral structural inhomogeneities.

Richards and Walker (1959), Cumming and Kanasevich (1966), and others agree that there exists another major seismic discontinuity below the Precambrian surface known locally as the Riel discontinuity, which possibly corres-

Figure 1.3a: The Western Canada Sedimentary Basin
(after the Alberta Society of Petroleum Geologists).

①

RIMBEY-MORINVILLE
Reef Chain



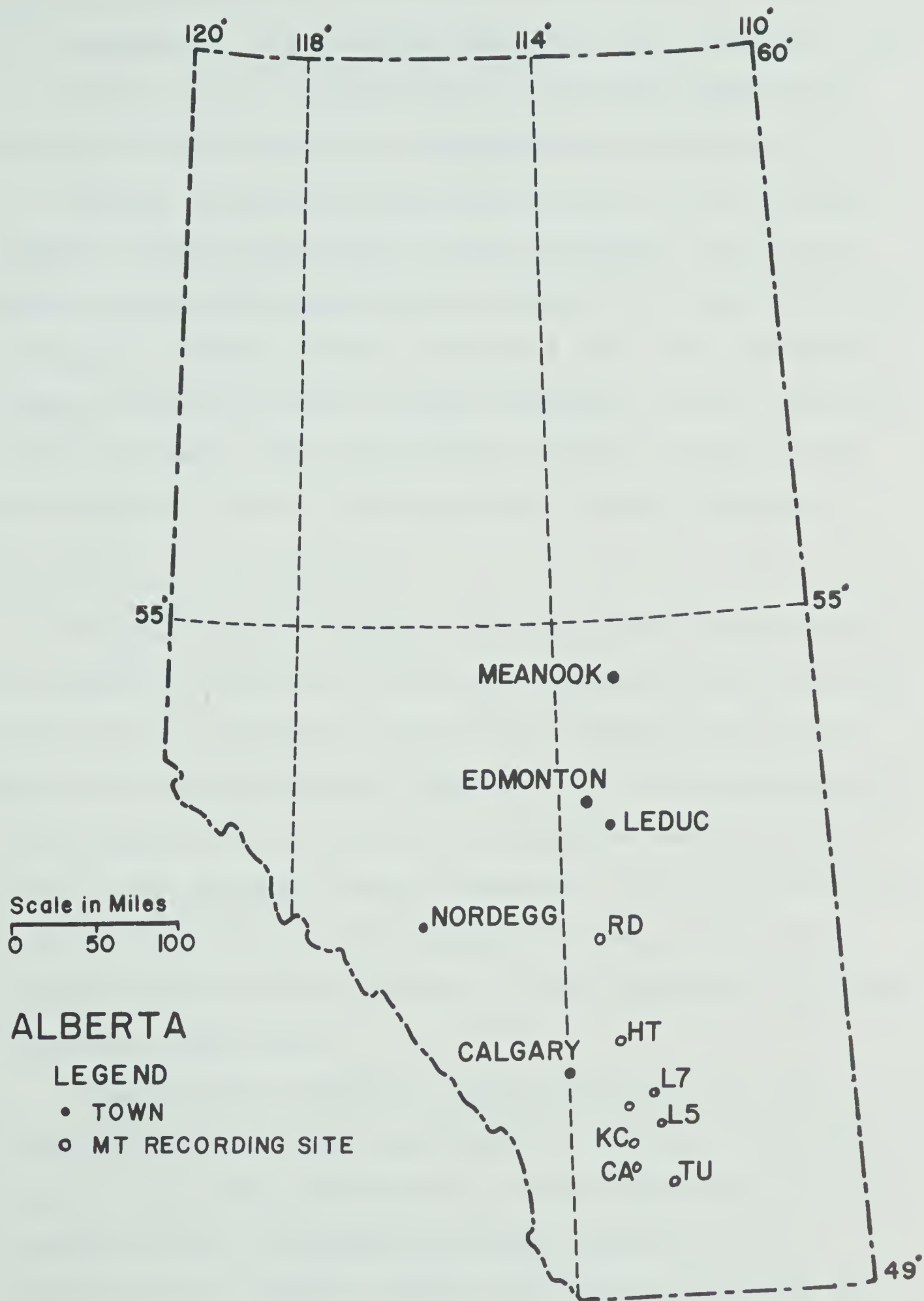
Regional	Geological	Cross	Section
of the			

WESTERN CANADA SEDIMENTARY BASIN

1' 10 200 mms (1000)
 1' 10 2000 mms (10000)

Reproduced through courtesy of Alberta Society of Petroleum Geologists

*Figure 1.3b: Locations of magnetotelluric field stations
and other regions of interest.*



ponds to the European discontinuity known as the Conrad. The Riel appears to vary in depth from 26 to 37 kilometers. The next major seismic discontinuity is the Mohorovičić which varies in depth from 38 to 47 kilometers in Alberta.

Clowes, Kanasawich and Cumming (1968) have used deep crustal seismic reflections at near vertical incidence to detect an elongated precambrian graben-horst structure (rift valley) in southern Alberta (Figure 4.3c). This structure seems to coincide with a notable anomalous linear trend in gravity (Figure 4.3a) and magnetic surveys (Figure 4.3b) which extends across Alberta and into British Columbia.

1.4 Outline of the Thesis

Chapter two will present the theoretical development of the apparent resistivity function, assuming plane wave incidence for an isotropic, horizontally layered earth model. The basic magnetotelluric impedance tensor relationships for a two-dimensional structure are introduced and the properties of the impedance tensor presented. This is followed by a short discussion of the effects of lateral resistivity changes and the effect of finite source dimensions on magnetotelluric quantities.

Chapter three contains a brief description of the recording equipment and the digital processing technique used for this study followed by a basic description of the procedure used to calculate the power spectra of the magnetotelluric fields, and how the spectra were used to

calculate the magnetotelluric impedance tensor.

In chapter four, the basic analyses for eight locations in southern Alberta at which magnetotelluric results were obtained are presented and correlated with other geophysical measurements. An explanation is offered for the ambiguous results obtained by other workers in Alberta and for the anisotropic resistivity properties which are consistent over nearly the entire province.

Chapter five contains the conclusions of this work and suggestions for further work.

CHAPTER II

MAGNETOTELLURIC THEORY

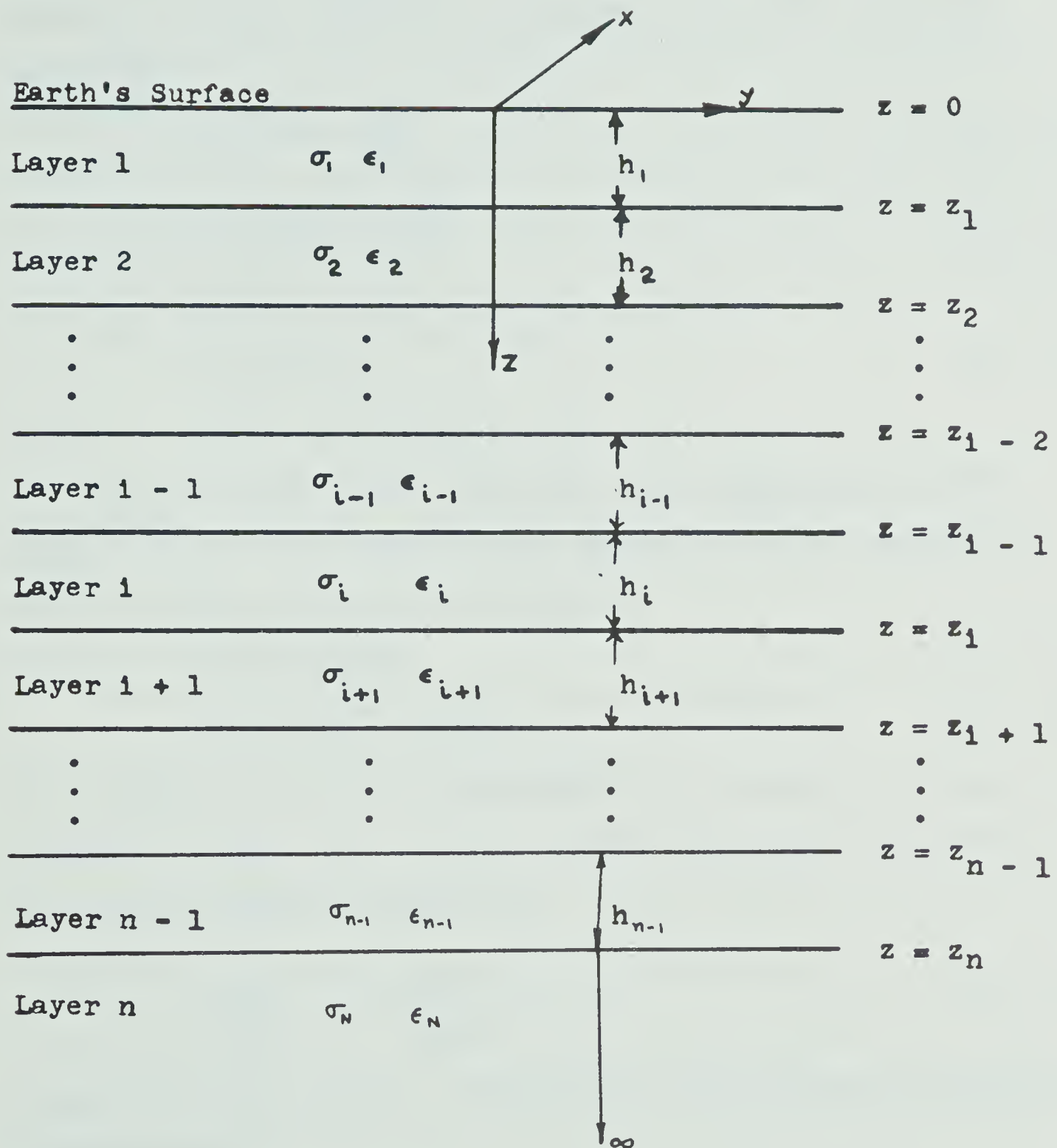
2.1 Basic Magnetotelluric Theory

The basic theory of magnetotelluric fields is contained in a boundary value problem involving Maxwell's equations and the resultant electromagnetic wave equation. The relationship between orthogonal components of the electric and magnetic fields measured at the surface of the earth are a measure of the electrical properties of the earth.

The reference coordinate frame for the theoretical development which follows is a right-handed cartesian system with the positive x axis northward and z vertically downward. The earth is initially considered to consist of $n - 1$ horizontally stratified, isotropic, homogeneous conducting layers above a basal half space. The origin of the coordinate system lies on the upper surface of the first layer as shown in Figure 2.1. The rationalized MKS system will be used in the theoretical discussion.

The natural electromagnetic disturbance fields are thought to originate in the ionosphere or magnetosphere at altitudes of 100 kilometers and greater and these fields then propagate downward to the surface of the earth. The similarity of the magnetotelluric field over large regions at the earth's surface was noticed by Schlumberger and Kunetz (1946) and Kunetz (1953); thus Cagnaird (1953) made

Figure 2.1: The horizontally stratified earth model used in the theoretical development of the apparent resistivity function.



the *a priori* assumption that the natural electromagnetic disturbances could be treated as plane waves.

Since the layers of the model are assumed to contain no primary sources, Maxwell's equations in the i^{th} layer are given by:

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad 2.1 - 1$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad 2.1 - 2$$

$$\nabla \cdot \vec{D} = \rho = 0 \quad 2.1 - 3$$

$$\nabla \cdot \vec{B} = 0 \quad 2.1 - 4$$

where $\vec{J} = \sigma \vec{E}$, $\vec{D} = \epsilon \vec{E}$, and $\vec{B} = \mu \vec{H}$.

For a steady state sinusoidal time dependence of the form $e^{-j\omega t}$ for the electric and magnetic fields, equations 2.1 - 1 and 2.1 - 2 can be written:

$$\nabla \times \vec{E} = j\omega\mu\vec{H} \quad 2.1 - 5$$

$$\nabla \times \vec{H} = \sigma\vec{E} - j\omega\epsilon\vec{E}. \quad 2.1 - 6$$

Taking the curl of equations 2.1 - 5 and 2.1 - 6 and assuming ϵ and σ are constant within each layer, we obtain two vector Helmholtz equations

$$\nabla^2 \begin{bmatrix} \vec{E} \\ \vec{H} \end{bmatrix} + k^2 \begin{bmatrix} \vec{E} \\ \vec{H} \end{bmatrix} = 0 \quad 2.1 - 7$$

where $k^2 = j\omega\sigma\mu + \epsilon\mu\omega^2$. Solutions of the Helmholtz equations are given by:

$$\begin{bmatrix} \vec{E} \\ \vec{H} \end{bmatrix} = \begin{bmatrix} \vec{E}_0 \\ \vec{H}_0 \end{bmatrix} e^{j\vec{K} \cdot \vec{r} - j\omega t} \quad 2.1 - 8$$

where

$$\vec{K} = (k_x, k_y, k_z,)$$

$$\vec{r} = (x, y, z).$$

The frequencies considered in magnetotelluric methods are nearly always less than 10 Hz. Using normal parameters of conductivity, permittivity, and permeability appropriate to materials composing the earth, the displacement currents are negligible in comparison to conduction currents for the low frequencies considered herein

$$|j\omega\sigma\mu| \gg |\epsilon\mu\omega^2|.$$

The Helmholtz equations then reduce to diffusion equations with a diffusion constant (skin depth) given by

$$\delta = \sqrt{\frac{2}{\mu\omega\sigma}}. \quad 2.1 - 9$$

Since the wave number of the electromagnetic field in air is much less than in the earth,

$$k_{\text{Air}}^2 = \epsilon_0 \mu_0 \omega^2 \ll |j\omega\mu\sigma| = k_{\text{Earth}}^2,$$

at the air-earth interface the incident plane waves are refracted and propagate through the earth nearly parallel to the vertical axis for nearly all incident angles.

For the geometry and conditions stated above, $\vec{E}$ and $\vec{H}$ are independent of x and y and thus all derivatives of the field with respect to x and y vanish. Thus, in the i^{th} layer a solution of equation 2.1 - 7 is given by

$$E_x^i = \left[A^i e^{jk_1 z} + B^i e^{-jk_1 z} \right] e^{-j\omega t} \quad 2.1 - 10$$

and from equation 2.1 - 5

$$H_y^i = \frac{+k_1}{\omega\mu} \left[A^i e^{jk_1 z} - B^i e^{-jk_1 z} \right] e^{-j\omega t}. \quad 2.1 - 11$$

For the simple case of a basal half space of uniform conductivity, the ratio of the electric field strength to the magnetic field strength can be defined as the characteristic plane wave impedance of the medium:

$$\begin{aligned} Z_c &= \frac{E}{H} = \frac{\omega\mu}{k_1} \cdot \frac{A e^{jk_1 z} + B e^{-jk_1 z}}{A e^{jk_1 z} - B e^{-jk_1 z}} \\ &= \frac{\omega\mu}{k_1} \coth \left[k_1 z + \ln \sqrt{\frac{A}{B}} \right]. \end{aligned} \quad 2.1 - 12$$

The reflection and transmission coefficients, A and B, are determined by the evaluation of the impedance at two different positions $z = z_1, z_2$ within the medium.

$$Z(z_1) = \frac{\omega\mu}{k_1} \coth \left[k_1 z_1 + \ln \sqrt{\frac{A}{B}} \right]$$

therefore:

$$\ln \sqrt{\frac{A}{B}} = \coth^{-1} \left[\frac{k_1}{\omega\mu} Z(z_1) \right] - k_1 z_1. \quad 2.1 - 13$$

At the second position in the medium:

$$Z(z_2) = \frac{\omega\mu}{k_1} \coth \left[k_1 (z_2 - z_1) + \coth^{-1} \left\{ \frac{k_1}{\omega\mu} Z(z_1) \right\} \right].$$

Letting $z_2 = 0$ and $z_1 \rightarrow \infty$:

$$Z_o(\omega) = \frac{\omega\mu}{k_1} = -[j\omega\mu\rho]^{1/2} \quad 2.1 - 14$$

and the resistivity of the medium is given by:

$$\rho(\omega) = \frac{1}{j\omega\mu} Z_O(\omega) = \frac{1}{j\omega\mu} \left| \frac{E_x}{H_y} \right|^2. \quad 2.1 - 15$$

For an n - layer electrically isotropic earth, both the electric field, E_x , and the magnetic field H_y , must be continuous across the boundaries between layers and therefore the impedance at the top of layer i must be equal to the impedance at the bottom of the next upper layer, $i - 1$. The impedance measured at the surface of the earth can therefore be written as (Berdichevski - 1960):

$$\begin{aligned} Z_O^n(\omega) = \frac{-\omega\mu}{k_1} \coth \left[k_1 h_1 + \coth^{-1} \left\{ \frac{k_1}{h_2} \coth (k_2 h_2 \right. \right. \\ \left. \left. + \coth^{-1} \left\langle \frac{k_2}{k_3} \cdots \coth^{-1} \left(\frac{k_{n-2}}{k_{n-3}} \coth (\right. \right. \right. \\ \left. \left. \left. k_{n-1} h_{n-1} + \coth^{-1} \frac{k_n}{k_{n-1}} \right) \cdots \right\rangle \right\} \right] \end{aligned} \quad 2.1 - 16$$

and the apparent resistivity of the medium can now be defined as:

$$\rho_a(\omega) = \frac{1}{\omega\mu} \left| Z_O^n(\omega) \right|^2. \quad 2.1 - 17$$

The apparent resistivity thus represents a weighted resistivity value for all layers and is the resistivity of an equivalent homogeneous half space at a particular frequency.

2.2 The Basic Magnetotelluric Relationships for an Electrically Anisotropic Earth

The importance of considering anisotropic conductivity

in models has been emphasized by Cantwell (1960), Bostick and Smith (1962), Ward and Morrison (1966) and others. Mann (1965) gave an analytical description of the effect of a plane wave incident on a basal half space with anisotropic electrical properties. This work was extended by O'Brien and Morrison (1967) to consider plane wave incidence on a horizontally layered medium in which each layer was electrically anisotropic and the principal resistivity axis of each layer aligned in an arbitrary horizontal direction.

The works of both Mann and O'Brien and Morrison have shown that if the wave impedance is defined as the scalar ratio of the electric field strength to the orthogonal magnetic field strength then the impedance is highly dependent on changes in the polarization of the disturbance fields.

When the conditions of model anisotropy and field polarization changes are considered, the relationship between the telluric and geomagnetic field can be expressed as a tensor:

$$\begin{vmatrix} E_x \\ E_y \end{vmatrix} = \begin{vmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{vmatrix} \cdot \begin{vmatrix} H_x \\ H_y \end{vmatrix} \quad 2.2 - 1$$

The mathematically equivalent admittance formulation introduced by Cantwell in 1960 can also be used. In the impedance formulation, the electric field is assumed to depend on both the parallel and the perpendicular magnetic fields. Changes in polarization of the source magnetic

field should have no effect on the tensor elements if the source field is a plane or quasi-plane wave.

The off diagonal elements ($z_{ij}; i \neq j$) of the impedance tensor can now be used to interpret geological structure if the principal axes of a two dimensional conductivity structure are aligned with the coordinate measuring axes. If the measuring axes are not aligned in the directions of principal conductivity then the tensor elements can be rotated computationally as follows.

Consider a two dimensional Cartesian coordinate system (x, y). When the system is rotated clockwise through some angle, θ , to form a new orthogonal system (x', y'), the transformed field components are given by:

$$\begin{aligned}\vec{E}'(x', y') &= \gamma \vec{E}(x, y) \\ \vec{H}'(x', y') &= \gamma \vec{H}(x, y)\end{aligned}\tag{2.2 - 2}$$

where the orthogonal transformation matrix, γ , is given by

$$\gamma = \begin{vmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{vmatrix}.\tag{2.2 - 3}$$

The rotated impedance tensor, Z' , is given by

$$Z' = \gamma Z \gamma^T.\tag{2.2 - 4}$$

For an isotropic and homogeneous earth

$$z'_{11} = z'_{22} = 0$$

and

$$z'_{21} = -z'_{12}\tag{2.2 - 5}$$

for all θ .

Analysis of anisotropic layered models in which plane wave incidence is assumed reveals the existence of two skin depths, and two modes of propagation for plane waves for any anisotropic layer of the model. These two modes are independent and propagate in the positive and negative z direction. Thus for a model with a single layer anisotropy, or a multi-layer anisotropy in which the principal directions of conductivity are aligned, there exists a rotation angle, θ , such that the rotated tensor elements exhibit the properties:

$$\begin{aligned} z'_{11} &= z'_{22} = 0 \\ z'_{12} &\neq z'_{21} \end{aligned} \quad 2.2 - 6$$

and it is valid to treat the resistivities in the two principal directions of anisotropy of the model as two separate isotropic resistivity systems.

To determine whether a two dimensional resistivity interpretation is possible, consider equation 2.2 - 4. It is easily shown that the following expressions are invariant under an orthogonal transformation:

$$z'_{11} + z'_{22} = z_{11} + z_{22}$$

and

$$z'_{12} - z'_{21} = z_{12} - z_{21}.$$

2.2 - 7

Since $z_{11} + z_{22}$ should vanish for the ideal two dimensional geometry, the ratio

$$\text{SKEW} = \frac{z'_{11} + z'_{22}}{z'_{12} - z'_{21}}$$

measures the skewness of the impedance tensor and thus provides a measurable two dimensionality coefficient.

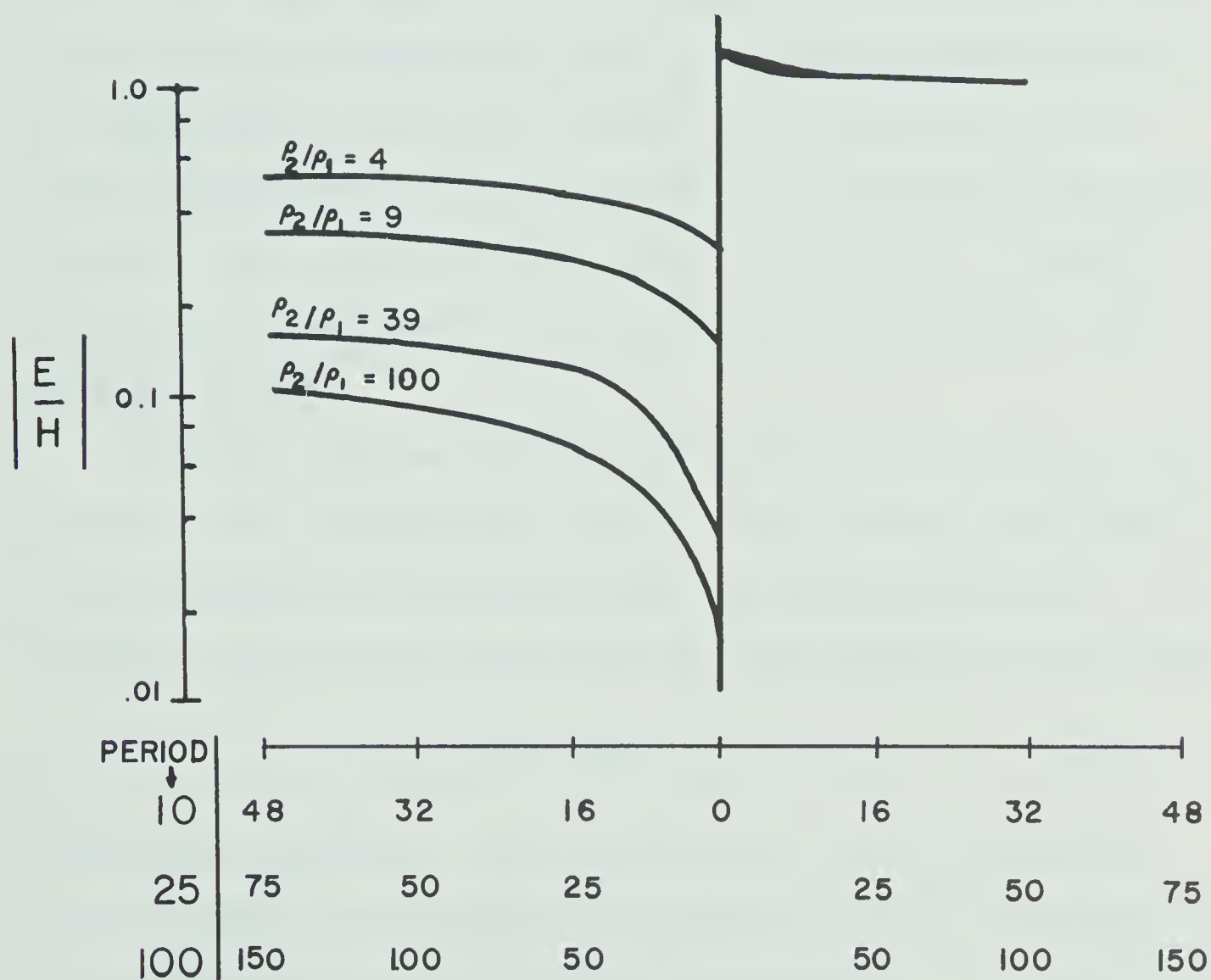
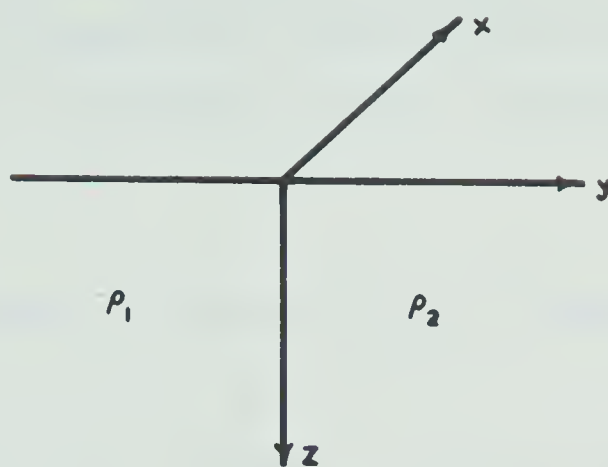
2.3 The Effects of Lateral Inhomogeneity

The addition of lateral resistivity changes to models representative of the earth, while being a better representation of some of the actual geophysical situations encountered, nevertheless leads to great mathematical difficulties. For this reason the exact solution of even a highly idealized model with laterally changing resistivity is of great benefit.

D'Erceville and Kunetz (1962) have analytically solved the problem of a vertical contact of infinite length separating two formations of resistivities ρ_1 and ρ_2 as shown in Figure 2.3a. Their solution considered the case where the magnetic field, $\vec{H}$, is parallel to the fault and constant at the air-earth interface ($z = 0$) of the model. The mathematical details of their solution will not be reproduced here as they are readily available in the literature.

Figure 2.3a shows the results of d'Erceville and Kunetz's solution for various resistivity ratios ($\frac{\rho_2}{\rho_1} = 4, 9, 39, 100$) across the contact. As can be seen, only at distances greater than one skin depth does the $|\frac{E_x}{H}|$ ratio approach the value of the non-fault model. This implies that for any location in which a large lateral resistivity contrast of sufficient spatial extent occurs, the apparent

*Figure 2.3a: Variations of the $|\frac{E}{H}|$ ratio for an infinitely deep vertical contact fault where the electric polarization is normal to the strike of the fault.
(After d'Erceville and Kunetz - 1962)*



HORIZONTAL SCALE (KM) FOR VARIOUS PERIODS

($\rho_1 = 400 \Omega \cdot M$)

resistivity function is biased by the fault structure and an accurate estimation of subsurface resistivities by the magnetotelluric method is doubtful unless a regional survey is made.

2.4 The Effect of Sources of Finite Size on Magnetotelluric Analysis

There exist no physical sources in nature which produce waves which are completely describable as mathematical plane waves. However valid the plane wave approximation may be for many cases of physical interest the effect of finite source dimensions on the magnetotelluric apparent resistivity function must be considered. This effect will be examined following the arguments of Wait (1954), Price (1962) and Morrison (1967).

Consider the two dimensional problem of a source above a horizontally layered earth model. The electric and magnetic fields produced by the source can be represented by an integral solution of the separated wave equation of the form:

$$\psi = \int_{-\infty}^{\infty} f(\lambda) e^{j\lambda x} e^{j\alpha z} d\lambda \quad 2.4 - 1$$

(Morse and Feshback - 1953) where the λ 's are associated with an effective propagation constant in the x direction and the effective horizontal spatial wavelength, λ_h , is given by:

$$\lambda_h = 2\pi/\lambda. \quad 2.4 - 2$$

The horizontal gradient of the field, $\frac{\partial \psi}{\partial x}$, gives a measure of the horizontal spatial changes of the field and therefore a horizontal scaling length, δ_h , can also be defined.

$$\delta_h = 1/\lambda \quad 2.4 - 3$$

where δ_h is the horizontal distance over which the field varies appreciably.

If the source had generated a plane wave which is incident upon the earth at an arbitrary angle θ , then λ would be associated with $k_o \sin \theta$. Since λ is a constant which characterizes the field in free space, let us denote λ as:

$$\lambda = k_o s \quad 2.4 - 4$$

where k_o is the propagation constant in free space. The horizontal scaling length is then expressed as:

$$\delta_h = 1/k_o s. \quad 2.4 - 5$$

Wait (1962) has shown that the surface impedance at the i^{th} layer may be expressed in terms of the impedance of the $i - 1$ interface and the thickness and parameters of the layer as:

$$z_{i+1} = \eta_{i+1} \frac{z_i - \eta_{i+1} \tanh [j u_{i+1} h_{i+1}]}{\eta_{i+1} - z_i \tanh [j u_{i+1} h_{i+1}]} \quad 2.4 - 6$$

where

$$u_i = \sqrt{k_i^2 - \lambda^2} = \sqrt{\sigma_i \mu_o \omega - \omega^2 \mu \epsilon s^2}$$

and

$$\eta_i = \frac{-j u_i}{\sigma_i}.$$

It is easily shown that the impedance value at the surface of a basal half space may be expressed as:

$$Z_o = \frac{E}{H} = - \sqrt{\frac{\omega \mu_o}{j \sigma_1}} \left[1 + j \frac{\omega \epsilon}{\sigma_1} s^2 \right]^{\frac{1}{2}}. \quad 2.4 - 7$$

For any conductivity and frequency parameters of this model, the maximum value of s can be calculated that will not radically alter the plane wave impedance value. From the above equation it can be seen that if $s < 0.1 \sqrt{\sigma/\omega \epsilon}$, the plane wave impedance is changed by less than 10% for a basal half space.

When the impedance value changes by 10% from the plane wave impedance, let s define the critical horizontal scale length:

$$L_c = 1/k_o s. \quad 2.4 - 8$$

Morrison (1967) has used this technique to calculate the critical lengths (Figure 2.4a) for four models shown in Figure 2.4b.

As an example of a physically realizable source, consider a line current flowing in the y direction at a constant height, $z = -h$, above a layered earth. The field produced

Figure 2.4a: Critical length versus period for four models described in figure 2.4b (after Morrison - 1967).

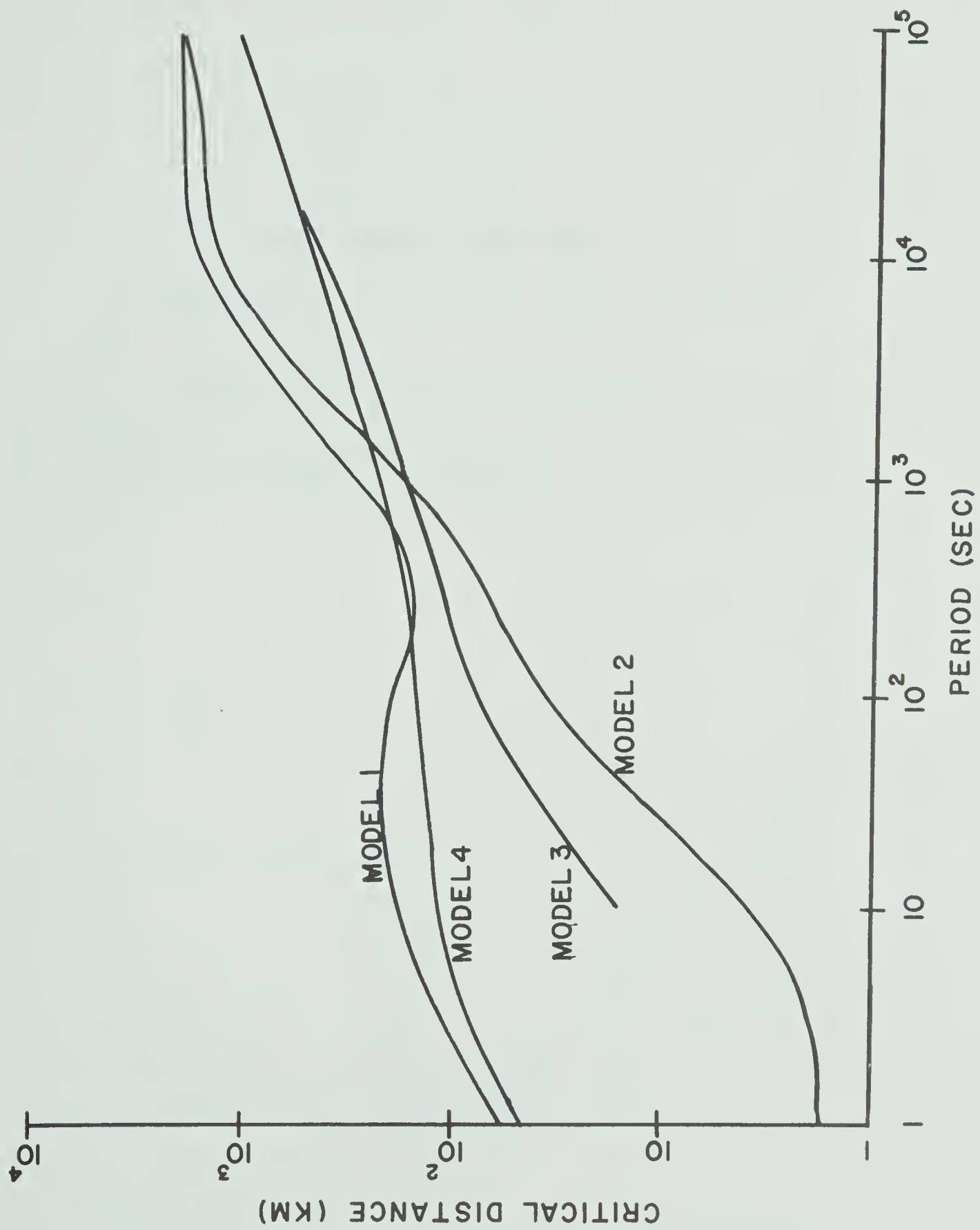


Figure 2.4b: Four resistivity models of the earth:

*Models 1 and 2: Fournier, Ward and Morrison
Model (1963)*

*Model 3: Modified Cantwell - McDonald Model
(Madden and Nelson - 1964)*

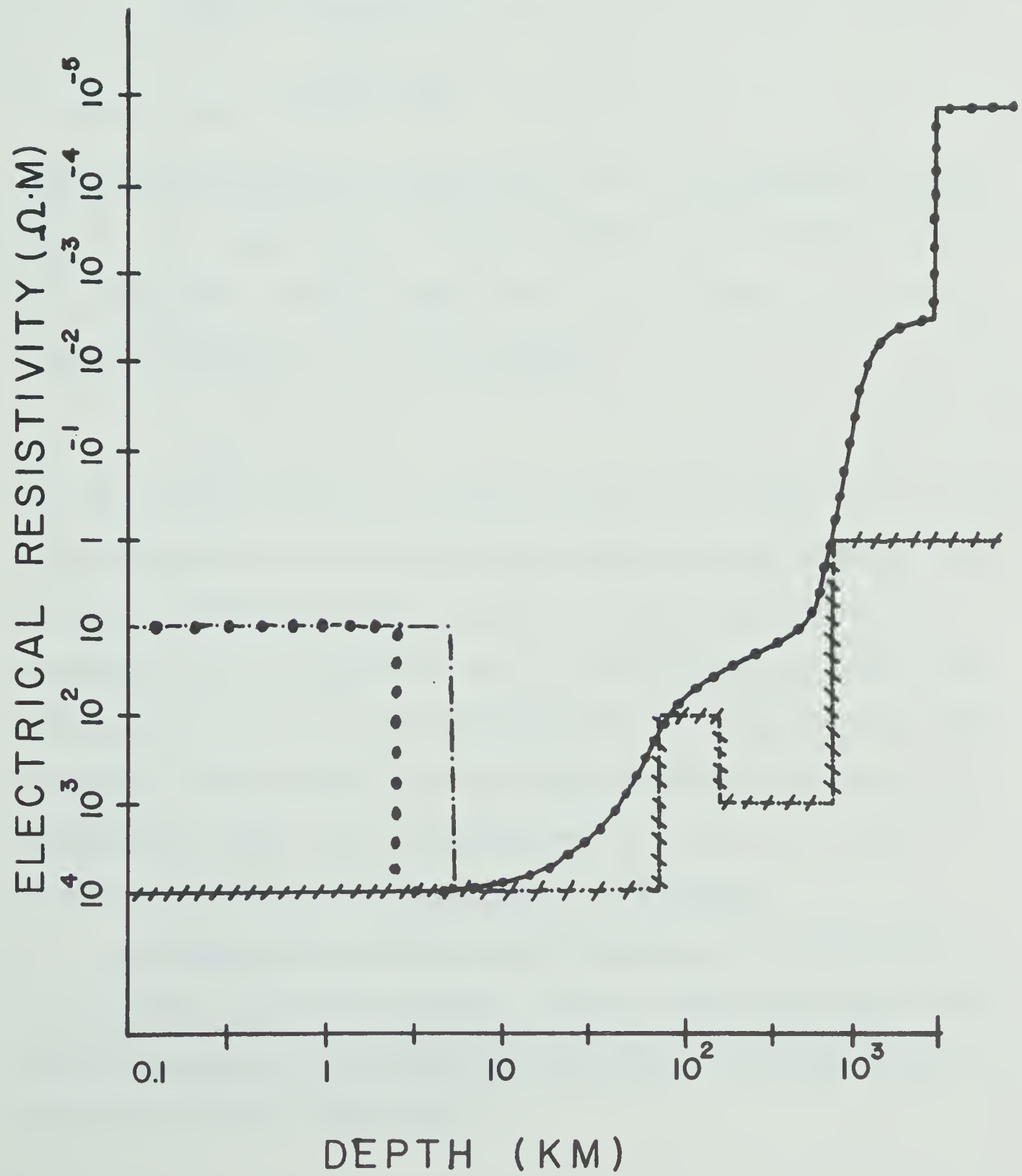
*Model 4: Cantwell - McDonald Model (Cantwell -
1960, McDonald - 1957)*

MODEL 1 . — . —

MODEL 2 ////

MODEL 3 • • •

MODEL 4 —



by the line source as measured at the surface of the earth is given by:

$$E_Y = A \int_{-\infty}^{\infty} \frac{e^{-j u_o h} [1 + R(\lambda)]}{u_o} e^{-j \lambda x} d\lambda \quad 2.4 - 9$$

(Wait - 1954) where $(1 + R(\lambda))$ accounts for the presence of the medium at $z = 0$ and

$$R(\lambda) = \frac{u_o - u_i}{u_o + u_i}$$

$$u_i = \sqrt{k_i^2 - \lambda^2}.$$

Since the integrand of equation 2.3 - 9 decreases rapidly for $\lambda > k_o$, then $u_o \approx j\lambda$ and $(e^{-j u_o h})/u_o \rightarrow e^{-\lambda h}/\lambda$, thus the integrand becomes small for $\lambda > 1/h$, and the critical length (equation 2.4 - 8) becomes:

$$L_c = 1/\lambda \approx h. \quad 2.4 - 10$$

We may assume therefore that the critical length for a line current source is approximately equal to the height of the current system above the earth. If the line current is located at an altitude of 100 - 200 kilometers, then from Figure 2.4a we see that magnetotelluric analysis will only be valid for periods less than 500 seconds if we are studying a model with a highly conducting first layer similar to the sedimentary basin of Alberta.

Although other source configurations may exist and invalidate the above analysis, this formulation does show the necessity for considering the effect of sources on magnetotelluric quantities.

CHAPTER III

DETECTION AND ANALYSIS OF THE MAGNETOTELLURIC FIELD

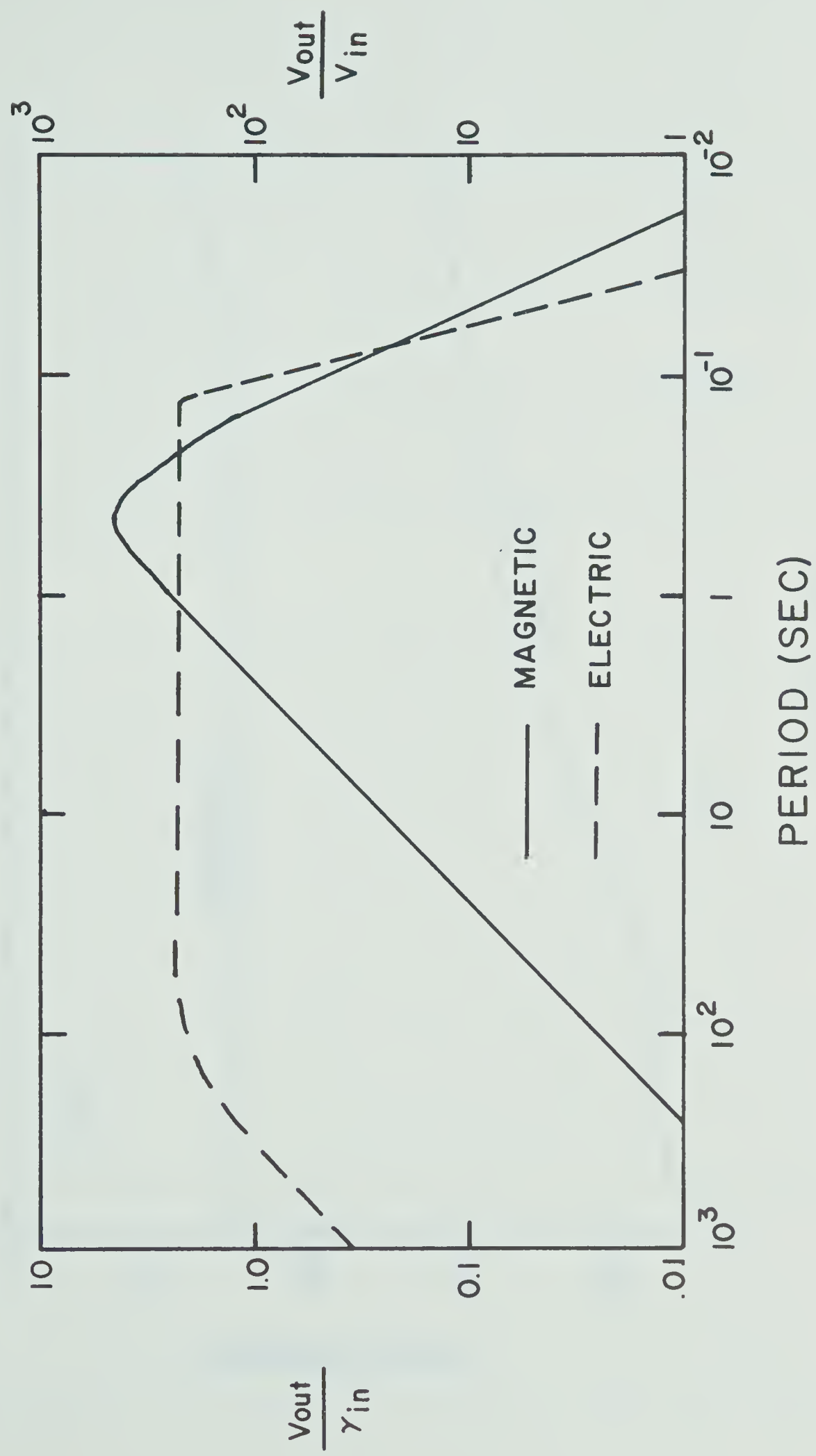
3.1 Detection and Recording of the Magnetotelluric Signal

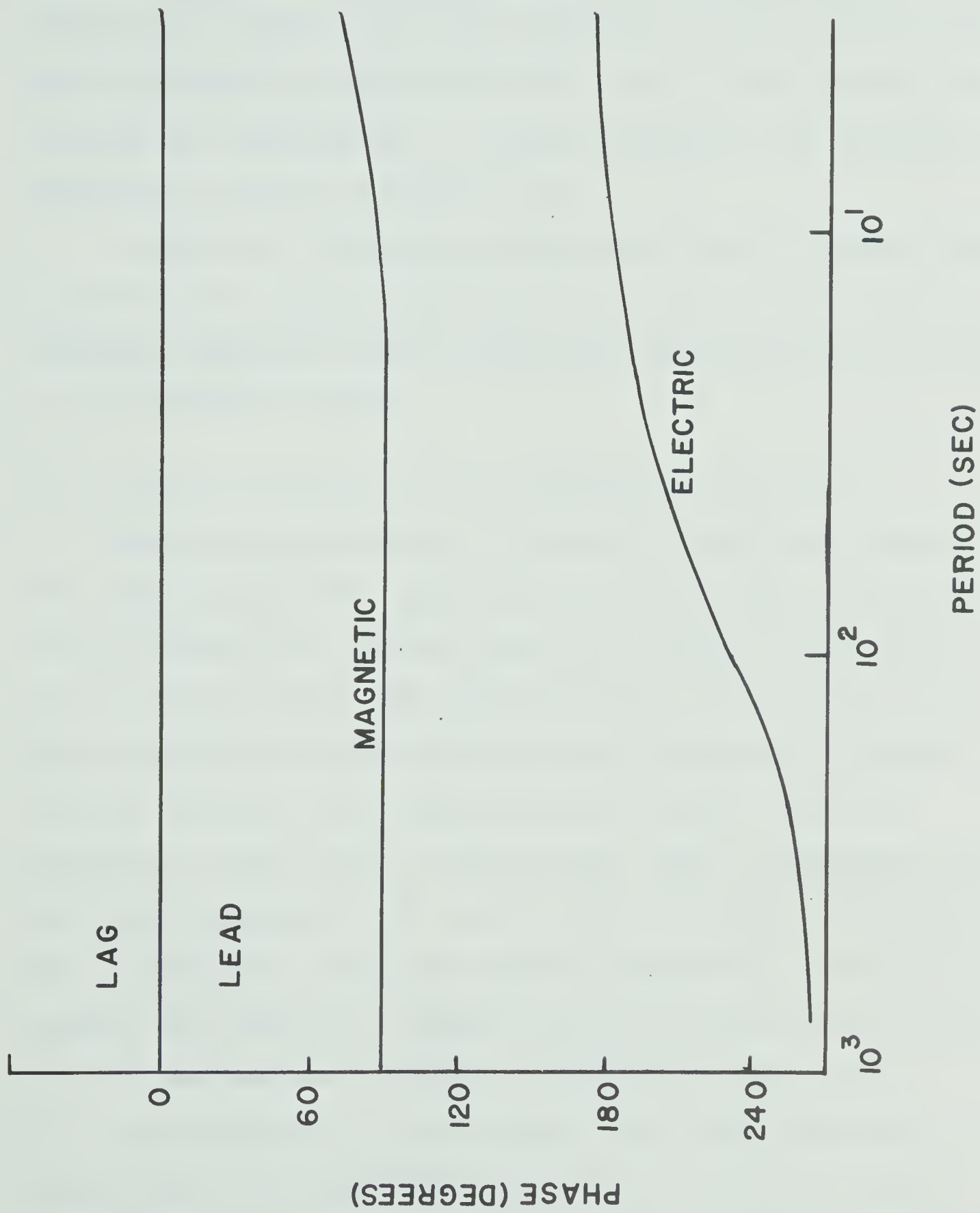
The telluric detection system consists of two inch copper pipe used as electrodes with the electrode separation approximately 600 feet (this separation distance varied from location to location due to physical limitations imposed by the area). The electrical potential difference between electrodes was fed through a 100 μ f capacitor to block D.C. potentials due to polarization at the electrodes and then into a filter/preamp/amp section.

The magnetic detection system consisted of two bobbins each wound with 16,000 turns of #28 GA enameled copper wire and then mounted on a 60 x 3/4 inch high permeability core. The total inductance was 850 henrys at 0.1 Hertz with a total impedance of 845 Ω . This unit was then sealed in a waterproof plastic tube. The current generated by the coil was fed to a galvanometer, and light beam coupled to an amplifier section. The general form of the calibrated responses of the telluric and magnetic systems (including coils) are shown in Figure 3.1a.

The output of the two telluric detection systems (Ex/ geographic north-south; Ey/ geographic east-west) and the three magnetic detection systems (Hx/ geographic north-south; Hy/ geographic east-west; and Hz/ vertical component)

Figure 3.1a: Calibrated phase and amplitude response for the telluric and magnetic detection systems.





along with WWVB time signals were recorded in analogue form on seven channel FM magnetic tape at 15/16 i.p.s. using a Precision Instruments tape recorder. Simultaneously, a Brush paper recorder was used to monitor visually the signals and to provide an editing facility prior to the selection of records for digitization. A block diagram of the recording apparatus is shown in Figure 3.1b.

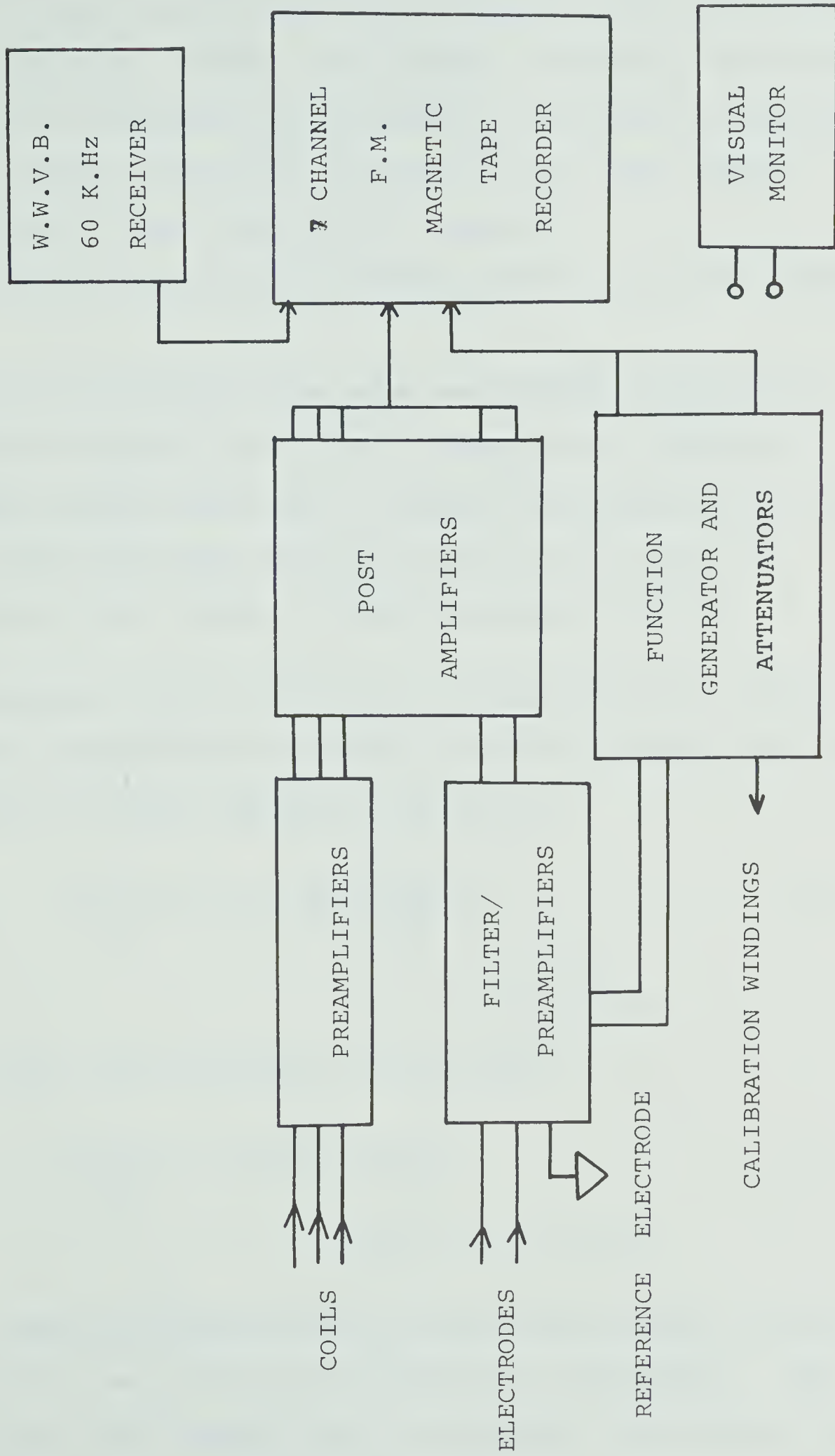
A series of calibration frequencies was fed through the system at the beginning and end of each twelve hour tape to provide a constant check on the phase and amplitude stability of the detection system.

3.2 Digital Processing of the Magnetotelluric Data

Records approximately 3.6 hours in length were selected for digitization from those analogue field tapes which were free of saturation (signal beyond the dynamic range of the FM recording system) and discontinuities (D.C. steps, etc.). The selection was based on the visual inspection of paper records obtained from replaying the FM tapes. The six channels of data (H_x , H_y , H_z , E_x , E_y , Time) in analogue form were then replayed at 30 i.p.s., alias filtered, and sampled by an electronic analog to digital converter at a rate of 15 samples per second per channel to give a digitizing interval of $2.1\bar{3}$ seconds and a Nyquist frequency of $.234\ldots$ Hz.

Digital data in 12 bit binary form (with the least significant bit suppressed and used as a flag for channel identification) were recorded in blocks of 1200 two character

Figure 3.1b: A block diagram of the magnetotelluric recording system.



words on six channel asynchronous Kennedy seven track digital tape recorders. Blocks were written alternately on digital tapes by two Kennedy tape recorders to provide the necessary I.R.G. (inter record gap) required for the tape reading facilities of the I.B.M. 360/67 computer.

A block diagram of the above system is shown in Figure 3.2a.

The two Kennedy tapes were not FORTRAN compatible on available computer facilities. Consequently a machine language program was used to transfer the data to nine track tapes with the proper computer word size (32 bits) for analysis by the author's FORTRAN language programs.

3.3 Spectral Density Estimates of Magnetotelluric Fields

The magnetotelluric method consists of calculating the Cagnaird or scalar apparent resistivity:

$$\rho_{a_{ij}}^{(s)}(\omega) = 0.2 \frac{2\pi}{\omega} \left| \frac{E_i(\omega)}{H_j(\omega)} \right|^2 \quad 3.3 - 1$$

$$i, j = 1, 2; i \neq j$$

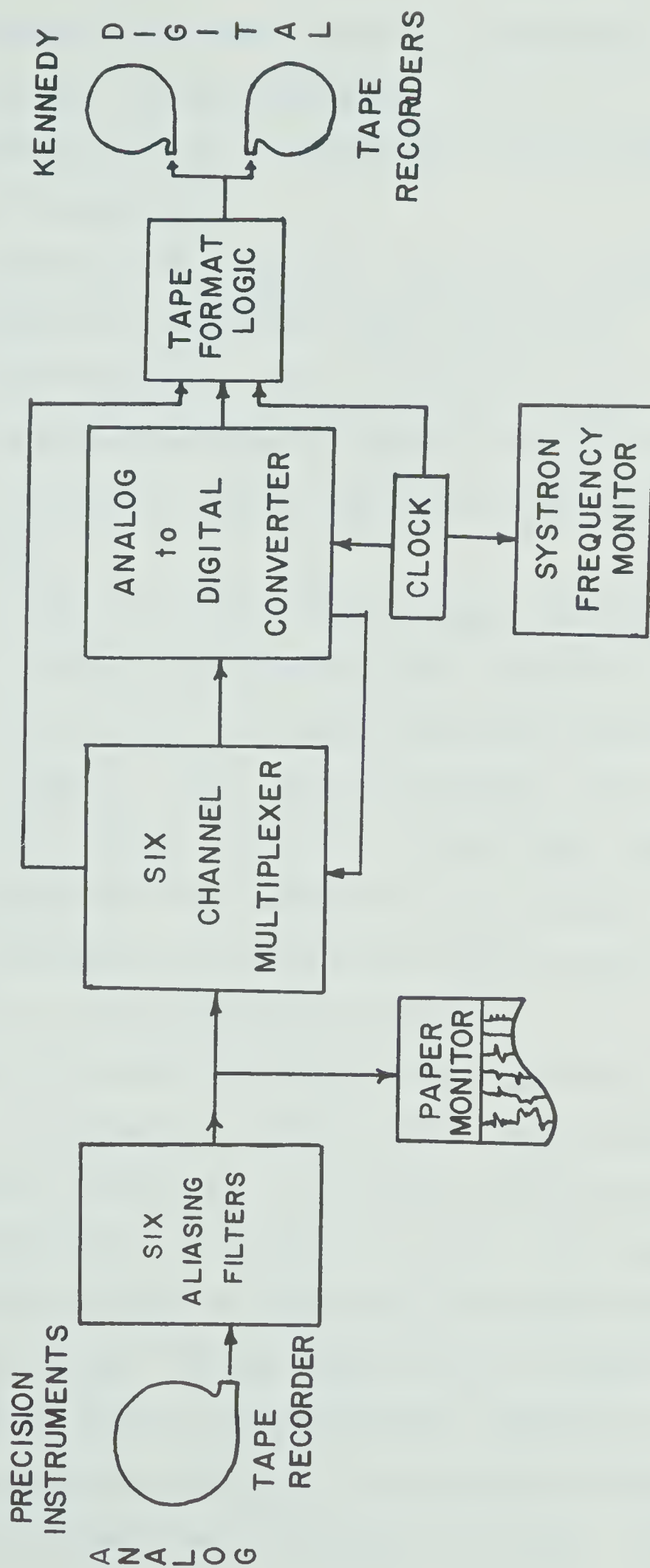
and/or the tensor apparent resistivity:

$$\rho_{a_{ij}}^{(z)}(\omega) = 0.2 \frac{2\pi}{\omega} \left| Z_{ij} \right|^2 \quad 3.3 - 2$$

$$i, j = 1, 2; i \neq j$$

for a measured magnetotelluric field and plotting the apparent resistivity as a function of period (or frequency). The curves are then matched with the apparent resistivity curves

Figure 3.2a: A block diagram of the analog to digital conversion system.



of various theoretical models until a reasonable fit to an appropriate model has been obtained.

The quantities $|E(\omega)|^2$ and $|H(\omega)|^2$ can be obtained by three general techniques:

- 1) Inspection of analogue records for sinusoidal events to obtain the amplitude values (Berdichevsky - 1960; Srivastava et al - 1963).
- 2) Filtering analogue or digital records for sinusoidal or quasi-sinusoidal events (Morrison - 1967).
- 3) Calculation of the digital power spectrum estimates by considering the records to be a sample from a stationary stochastic process (Cantwell - 1960; Vozoff, Hasagawa, and Ellis - 1964; Hopkins and Smith - 1966; and others).

The third method has been adopted for this study since the electronic processing of field data allows large quantities of information to be processed efficiently.

The power spectrum estimates were calculated using the techniques of Blackman and Tukey (1958) (calculating the auto-and-cross covariances) allied with the Fast Fourier Transform Algorithm (Cooley-Tukey - 1965). The covariance functions were obtained by convolution theorem techniques (Gentleman and Sande - 1965) rather than by summing the lagged products which is computationally expensive.

Detailed mathematical proofs of the procedures which follow are readily available (e.g. Blackman and Tukey - 1958) and

will not be presented here.

For analysis, data sets $X_i(k*\Delta)$, $X_j(k*\Delta)$ ($1 < k < N$; Δ = digitizing interval) were selected and zeros added to give the hyperseries:

$$X'_r(k*\Delta) = \begin{matrix} X_r(k*\Delta) & 1 < k < N; & r = i, j \\ 0 & N + 1 < k < N' \end{matrix}$$

where $(N' - N)$ is greater than the number of smoothed power spectrum estimates (M). The hyperseries were then discretely transformed to the frequency domain by an N' point Fourier transform. The resulting Fourier amplitudes, $F_i(\omega)$, $F_j(\omega)$ were squared to give unsmoothed periodogram estimates, $|F_{ij}(\omega)|^2$, which have a high variance.

Time series in general are characterized by random changes of frequencies, amplitudes and phases. Since a basic requirement of Fourier analysis involves the assumption of fixed frequencies, amplitudes, and phases, the Fourier results must be modified to take into account the random nature of the time series, that is, techniques are needed to reduce the variance of the spectral estimates. To achieve this reduction, the periodogram estimates were inverse Fourier transformed to the lag domain (Wiener - Kintchine theorem: Sande - 1965) by the Fast Fourier Transform and the auto- (or cross-) covariance function, $C_{ij}(\tau)$, was smoothed with a Parzen window (Parzen - 1961).

$$C'_{ij}(\tau) = W(\tau) \cdot C_{ij}(\tau)$$

where

$$W(\tau) = \begin{cases} 1 - 6(\tau/M)^2 + 6(|\tau|/M)^3 & |\tau| < M/2 \\ 2(1 - |\tau|/M)^3 & M/2 < |\tau| < M \\ 0 & |\tau| > M \end{cases}$$

and M is the maximum lag number. The power spectrum estimates were then obtained by transforming the smoothed covariances to the frequency domain by a $2M$ point Fourier transform.

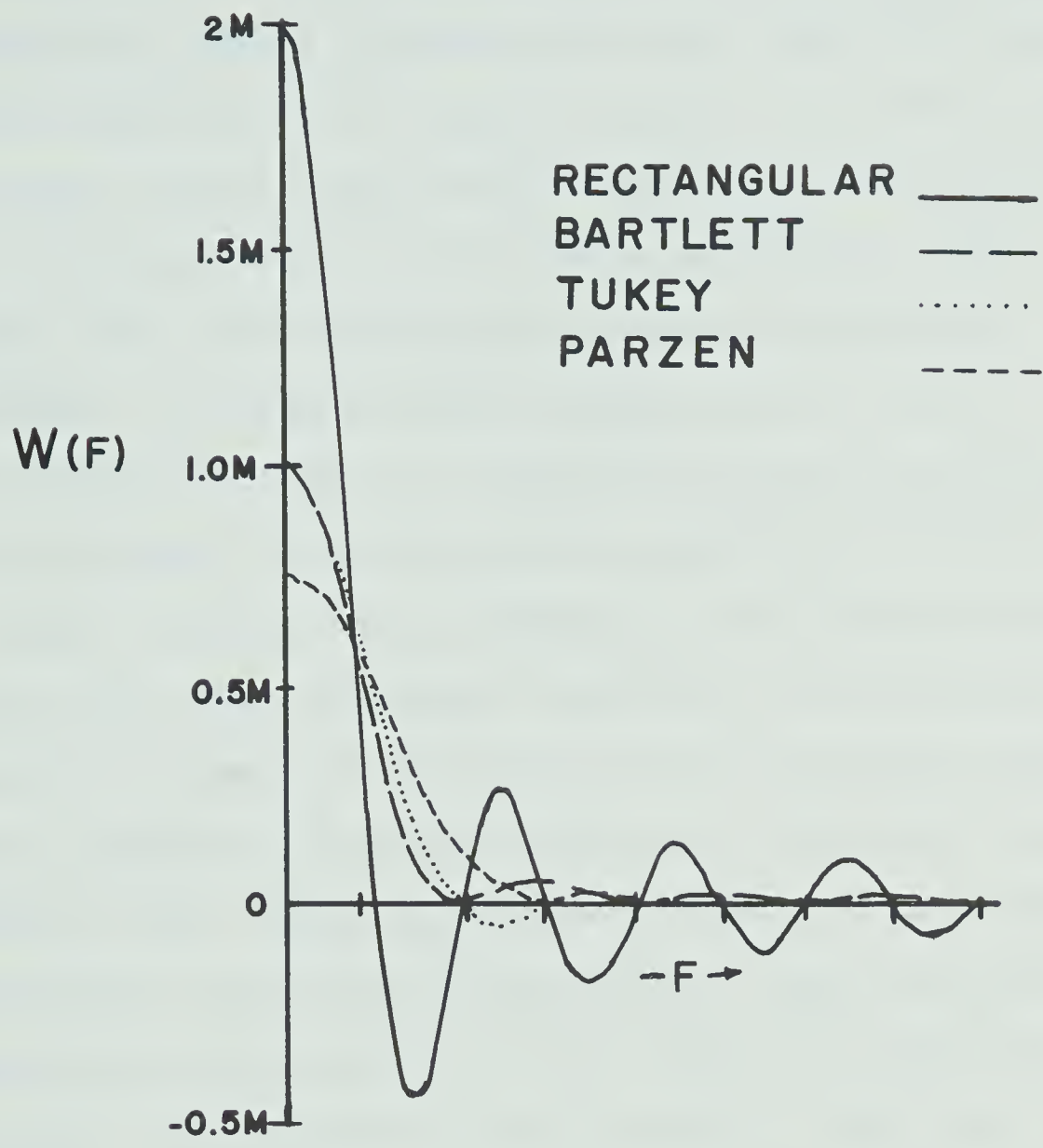
The Parzen lag window was chosen because it has a positive definite character, and achieves the smallest variance and greatest number of degrees of freedom of the windows tested. This was estimated to be of greater value than the negative effect of its relatively large bias seen in Figure 3.3a. The sample coherence function, $\gamma_{ij}(\omega)$, between data sets $X_i(k\Delta)$, and $X_j(k\Delta)$ was calculated by the usual definition

$$\gamma_{ij}(\omega) = \frac{P_{ij}(\omega)}{\sqrt{P_{ii}(\omega) * P_{jj}(\omega)}}$$

where $P_{ii}(\omega)$ and $P_{jj}(\omega)$ are auto power estimates and $P_{ij}(\omega)$ is the cross power estimate (in which the phase shift introduced by the digitizing process has been removed).

It was decided that the prewhitening technique would not be applied on the original data series as the instrument response (see Figure 3.1a) effectively prewhitens the spectra. To check this assumption, artificial time series were generated and various prewhitening windows were applied

Figure 3.3a: Common spectral windows.



but the prewhitening technique did not sufficiently enhance the reliability of the power spectrum estimates.

Although the above technique of computing spectral estimates was rather involved, the high speed and large storage capabilities of modern computers makes this process acceptable. Also, the Fourier transformation of two series at a time "in place" (see Gentlemen and Sande - 1966) reduces computer time and storage requirements considerably. These techniques, although slightly slower than the direct calculation and modification of the Fourier amplitude spectra, allow the direct inspection of the covariance function.

After testing, it was decided to use a minimum value of 0.90 for the sample coherency function as the criterion for acceptance of spectral estimates used to calculate the Cagnaird apparent resistivity function. Although this choice produced a high rejection rate of estimates, the effect produced was a very high reliability of power spectrum estimates and small scatter of the apparent resistivity estimates. Furthermore the choice of $N = 3,000$, $M = 256$ gave each spectral density estimate 42 degrees of freedom (Jenkins and Watts - 1968) which increased the statistical confidence of the final results to very acceptable levels.

3.4 The Estimation of the Impedance Tensor Elements

The basic impedance relationship can be written:

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} H_x \\ H_y \end{bmatrix} \quad 3.4 - 1$$

or

$$\begin{aligned} E_x(\omega) &= z_{11} H_x(\omega) + z_{12} H_y(\omega) \\ E_y(\omega) &= z_{21} H_x(\omega) + z_{22} H_y(\omega) \end{aligned} \quad 3.4 - 2$$

Post-multiplying the above equations by $H^*x(\omega)$, $H^*y(\omega)$, $E^*x(\omega)$, $E^*y(\omega)$ (where * indicates complex conjugate) yields eight equations in four unknowns. Solving any two equations from this overdetermined system of equations simultaneously, six solutions for each impedance element can be obtained (see APPENDIX I). The techniques described in Section 3.3 were used to calculate the auto- and cross- power spectra required for the estimation of each magnetotelluric impedance element.

Since no completely satisfactory "coherency" function exists as a criterion for the validity of the various tensor element estimates, a quantitative technique was demanded for the rejection of estimates with an unsatisfactory signal to noise ratio. Swift (1967) used a coherency function based on the "predictability" of the E field, however a simpler method seems satisfactory. Assume that the magnetotelluric fields are unpolarized, that is, $\langle E_x \cdot E^*y \rangle$, $\langle H_x \cdot H^*y \rangle$, etc. are negligible (where $\langle \rangle$ denotes a cross power spectral estimate), then only four of the six possible solutions are stable (see APPENDIX I). If the magnetotelluric field is polarized, one of the four "unpolarized" estimates is unstable.

If incoherent noise exists in the recorded magnetic and electric signals then two of the above stable unpolarized tensor estimates will have the form:

$$z'_{ij} = z_{ij} \left(1 + \frac{E \text{ noise power}}{E \text{ signal power}} \right) \quad 3.4 - 3$$

where z'_{ij} is the computed estimate and z_{ij} is the actual tensor value. The remaining two stable unpolarized tensor estimates have the form:

$$z'_{ij} = z_{ij} / \left(1 + \frac{H \text{ noise power}}{H \text{ signal power}} \right). \quad 3.4 - 4$$

Thus two unpolarized solutions are biased high due to electric noise while two unpolarized solutions are biased low due to magnetic noise. By averaging the four estimates of each impedance element the biasing effect of incoherent noise on the estimation of the tensor element can be reduced if the incoherent noise has the same power level in both the electric and magnetic data.

To eliminate further scatter, a minimum signal strength criterion was developed. The power level that six cycles of data (with an amplitude twenty times greater than the minimum increment of the digitizing apparatus) would produce for a given record length at each Fourier frequency was calculated. Rejection of those frequency bands in which the magnetic signal was less than the arbitrary minimum power level reduces the scatter of the tensor apparent resistivities by effectively increasing the signal to noise ratio to more acceptable limits.

3.5 Programming Considerations.

After a thorough study of spectral analysis and

statistical communication theory, FORTRAN language programs were written to facilitate the analysis of magnetotelluric data. Using the Fast Fourier Transform algorithm and the techniques outlined in Section 3.3, calculations of the power spectra, orthogonal scalar resistivities, phases, and coherencies for the recorded magnetotelluric field data were performed. All digitized magnetotelluric data was processed in this manner, and displayed by means of a typewriter plotter utilizing graphing programs developed by the University of Alberta Computing Science Department. This program was later expanded to calculate the rotated scalar resistivities, coherencies and phases but the results of a rotated scalar type of analysis were never satisfactory.

This led to the development of a FORTRAN language program to calculate the impedance tensor elements of the magnetotelluric data. Tensor estimates were calculated, and the impedance tensor elements computationally rotated (equation 2.2 - 4) and graphing routines were written to display the tensor elements and resistivities on the line printer. The four averaged tensor elements, rotated by 10° increments from 0° - 90° and plotted as a function of period, were then inspected visually to ascertain the rotation angle producing the $\min |z_{11}(T, \theta)|$. The impedance tensor elements were then recalculated, rotated into the principal directions of resistivity computationally, and the tensor apparent resistivity function was calculated (equation 3.3 - 2). All data from one location was treated in this manner and

programs were written to display the maximum, minimum and average values of apparent resistivity using a CALCOMP plotter.

Another program was written to calculate and display the polarization characteristics (degree of polarization, polarization angle, and ellipticity) of the magnetic field using the techniques of Born and Wolf (1959) as outlined by Fowler et al (1967) and the results of the analysis were also displayed on the CALCOMP plotter (see APPENDIX II).

CHAPTER IV

A MAGNETOTELLURIC EXPERIMENT IN SOUTHERN ALBERTA

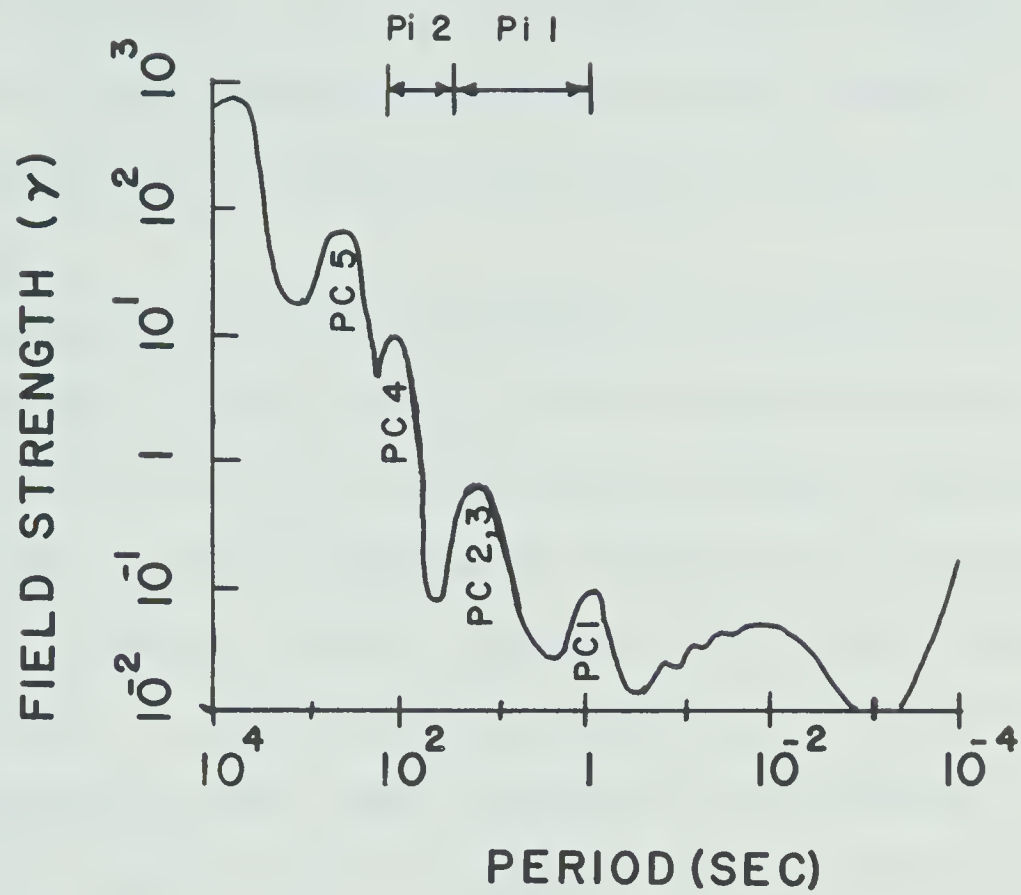
4.1 General Observations

During the summer of 1966 a magnetotelluric survey was made along a north-south profile in southern Alberta. In the summers of 1967-8, this profile was expanded and a second, three station profile made to facilitate a correlation of geologic structure between the two profiles. The locations of the stations comprising the profiles are shown in Figure 1.3b and this chapter will present the results of this survey.

The analysis of all data was made using the tensor techniques discussed in CHAPTER 3 and APPENDIX I and all computed quantities were calculated for record sections 6400 seconds long. For periods between 10 and 100 seconds, the time averaged impedance elements from various records of sufficient power show very little scatter. However for periods greater than 100 seconds the impedance elements are characterized by rather large variances. For all data presented in this chapter, the skew values of the magnetotelluric impedance were less than 0.3. This indicates that the sub-surface resistivity structure at all the stations could be reasonably represented by two dimensional models.

Figure 4.1a (Campbell - 1966) shows a very general diagram of field strength versus period for geomagnetic

Figure 4.1a: Identification of the geomagnetic micro-pulsation spectrum in the magnetotelluric frequency range (after Campbell - 1966).



Pulsation	Period
Pc 1	0.2-5.0
Pc 2	5.0-10.0
Pc 3	10- 45
Pc 4	45-150
Pc 5	150-600
Pi 1	1 - 40
Pi 2	40-150

micropulsations. Sharp dips in the spectrum (e.g. between the Pc4 and Pc5 bands) can decrease the reliability of the spectral analysis estimates and distort the apparent resistivity estimates. It is most desirable to obtain recordings of the magnetotelluric field which produce a spectrum containing a slowly varying power level at all frequencies.

To obtain the principal directions of resistivity, the tensor impedance elements were rotated computationally (equation 2.2 - 4) and the principal directions were defined by the angles, θ , which produced a minima in the absolute value of the rotated diagonal elements of the magnetotelluric impedance tensor. Although all frequency estimates of the rotated impedance tensor were calculated and examined, only three or four representative estimates will be presented in a diagram for each station. Several general observations can be immediately made.

- 1) Only in a few cases was $\min|z_{11}(T, \theta)|$ actually zero. This is expected due to noise contamination in the signal, source effects, a non-two dimensional resistivity structure, or to the fact that if the power spectra have sharp peaks, spectral analysis techniques will produce an averaged value of power over the band-width of the particular spectral window used.
- 2) The angle, θ , defined by $\min|z_{11}(T, \theta)|$ does not appear to be time invariant and variations as large as 25° in the rotation angle have been observed for the

same period in different data sets recorded at the same location. This is probably due to the inability of the spectral analysis techniques to reliably process data with steep power spectra slopes, since the estimation of θ is consistent from record to record if the power spectra was of sufficient activity and slowly varying.

For periods greater than 100 seconds, the magnetic field tends on the average to exhibit strong elliptical and linear polarization with the major axis of the polarization ellipse oriented between north and east. Graphs of representative magnetic polarization parameters and instrument prewhitened magnetic spectra for all stations are presented in APPENDIX II.

The tensor analysis of magnetotelluric data from all locations studied in southern Alberta indicated that the minor axis of resistivity aligned between N 40°E and N 80°E. This alignment of the principal direction of resistivity and the magnetic polarization ellipse meant that a much lower signal to noise ratio existed for one tensor apparent resistivity estimate than the other. This resulted in a relatively large scatter of the ρ_{a12}^z estimates as compared to the ρ_{a21}^z scatter.

4.2 Analysis of Magnetotelluric Data

Over twenty magnetotelluric recording locations have been occupied in central and southern Alberta. The analysis of results from eight of these stations will now be presented.

- i) Red Deer Location (RD) (52° 10' Latitude, 113° 27' longitude, elevation: +3100 feet)

This location was chosen to determine the apparent resistivity curves over the sedimentary basin of western Canada at a large distance from the seismically determined elongated horst-graben structure (rift valley) in southern Alberta. Garland and Burwash (1959) indicate that the Precambrian rock at the basement surface is gneiss at the magnetotelluric recording location but within 25 miles to the south and west the basement character is chiefly granitic (see Figure 4.4b).

Preliminary scalar resistivity analysis (Figure 4.2a) would indicate that the subsurface resistivity properties were reasonably isotropic. The scalar apparent resistivity curves displayed in Figure 4.2a are a composite of results from the analysis of seven data sets (each data set 6400 seconds long). The averaged apparent resistivity is indicated by a symbol and the error bars indicate the maximum scatter in apparent resistivity for estimates with a coherency greater than 0.90.

Tensor analysis of the seven data sets indicated however that the location has anisotropic resistivity properties for periods greater than ten seconds. The magnetotelluric impedance tensor indicated definite minima of the diagonal elements for the rotation angles $\theta_1 \approx N 40^\circ E$ and $\theta_2 \approx N 40^\circ W$ (as shown in Figure 4.2b) implying that the major and minor

directions of resistivity are approximately cartesianally orthogonal.

Skew values of the magnetotelluric impedance tensor were less than 0.3 for all estimates shown on the tensor apparent resistivity curves. For this reason, the subsurface resistivity structure cannot be three dimensional in character. The tensor apparent resistivity ρ_{21} curve has been modeled with a one dimensional isotropic layered model assuming plane wave incidence.

The basic surface features for the model of the $\rho_{21}(T, \theta)$ curves was derived from local well log data. This model does not appear to fit the Red Deer $\rho_{21}(T, \theta)$ curve nearly as well as it should if the resistivity structure were actually one dimensional. Figure 4.4b from Garland and Burwash (1959) indicates lateral inhomogenieties in the basement composition near this site and this may explain the deviation of the experimental curves from a one dimensional model curve. The effect of large scale distant structures will be discussed below.

Figure 4.2a: The scalar apparent resistivity curves from the Red Deer location.

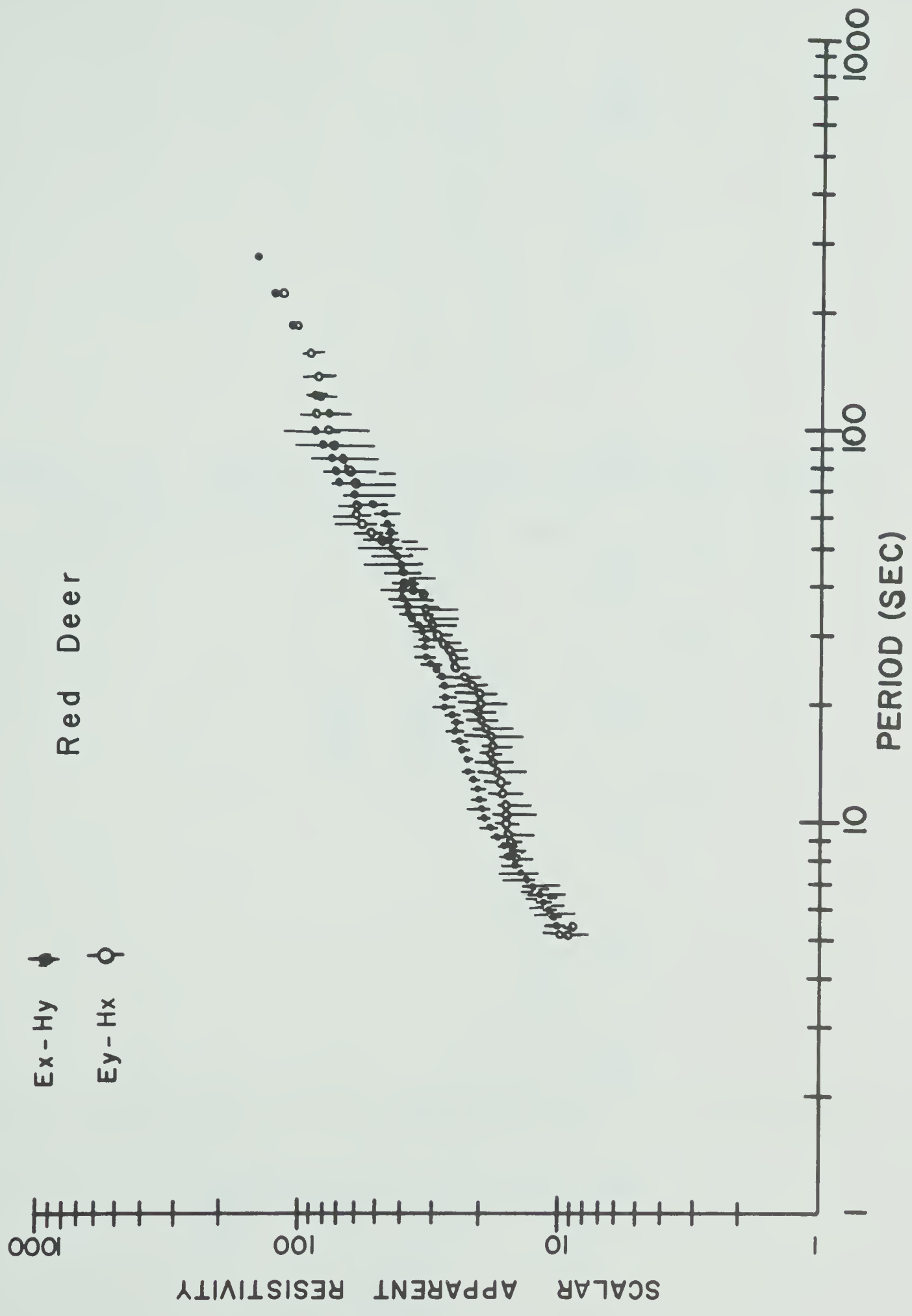
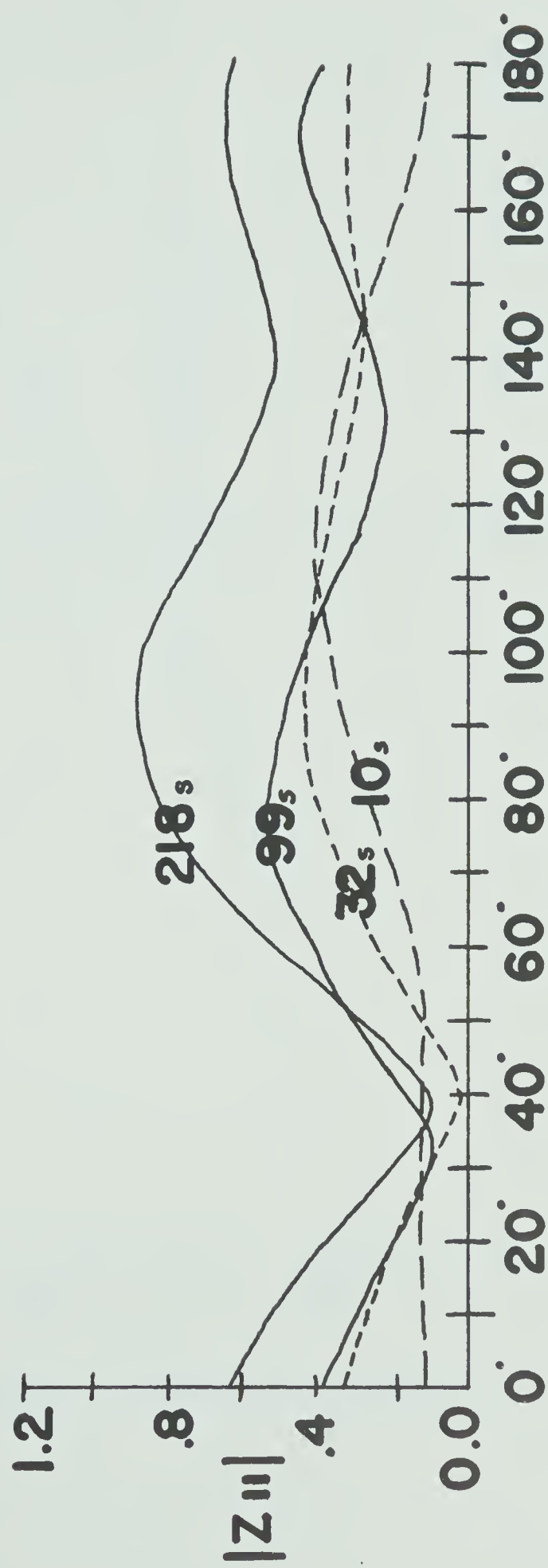


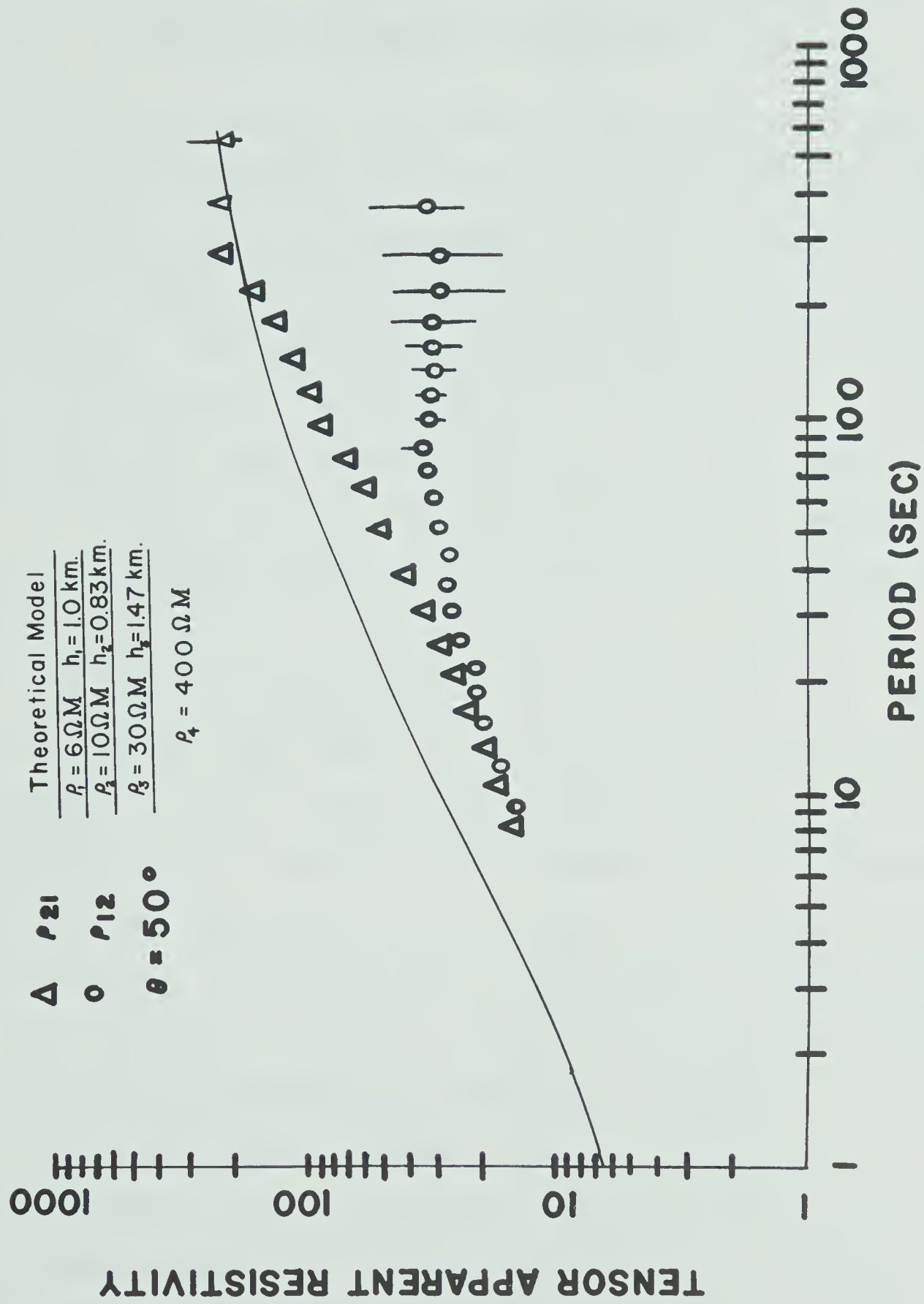
Figure 4.2b: Rotated diagonal tensor element at selected periods for Red Deer location data.



ROTATION ANGLE

RED DEER

Figure 4.2c: Tensor apparent resistivities for the Red Deer location.



- ii) Horse Thief Canyon (HT) ($51^{\circ} 34'$ latitude, $112^{\circ} 55'$ longitude, elevation: +2300 feet)

This location was chosen to represent the north end of both the expanded 1966 profile and the additional 1968 profile. The recording apparatus was situated on the floor of a canyon 400 feet below the regional surface level of the plains. The canyon forms a north-south trending rectangular channel which is approximately 13,000 feet wide. The Red Deer River runs north-south along the canyon floor.

A plot of $|z_{1,1}|$ versus rotation angle indicates that the principal directions of resistivity are defined by the rotation angles $\theta_1 = 90^{\circ} - 110^{\circ}$, and $\theta_2 = 160^{\circ} - 170^{\circ}$, for all periods investigated as shown in Figure 4.2d. This relatively high degree of noncartesian orthogonality (compared to other locations investigated) for the principal directions of resistivity is primarily due to topographical features superimposing their effect on the true resistivity features. The electrodes at this station were only 600 feet in length and therefore far less than ten times greater than the major topographical variations. Wescott and Hessler (1962) have made a telluric current model study of a rectangular channel and found that the measured electrical potential per unit length was greater in a rectangular channel than external to the channel region. As the width to height ratio of the rectangular channel increased the

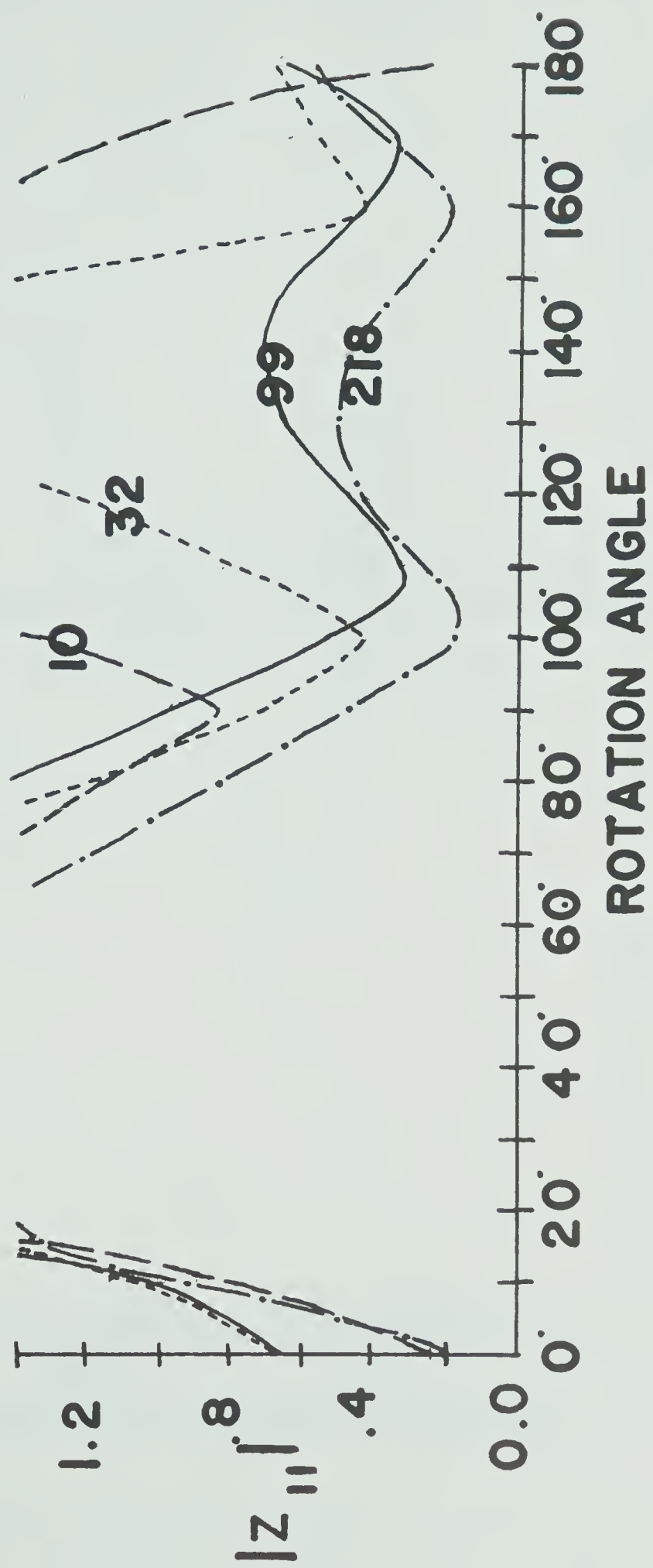
ratio of

$$\frac{\text{potential per unit length in channel}}{\text{potential per unit length outside channel}}$$

decreases to approximately one. The canyon has a width to height ratio of 33:1. Since the telluric (and magnetic) measurements were made entirely within the canyon, the measured apparent resistivities were biased by the shape of the canyon. The model shown in Figure 4.2d was derived from local well log data and again is a poor fit. Figure 4.4b (basement lithology) indicates lateral inhomogenities in the basement composition within 25 miles to the west of this location. This also contributes to the inability of a one dimensional model to adequately explain the form of the apparent resistivity curve. In addition to the large scale structural effect of the more distant Rocky Mountains the local surface structure effects the high frequency end of the curves.

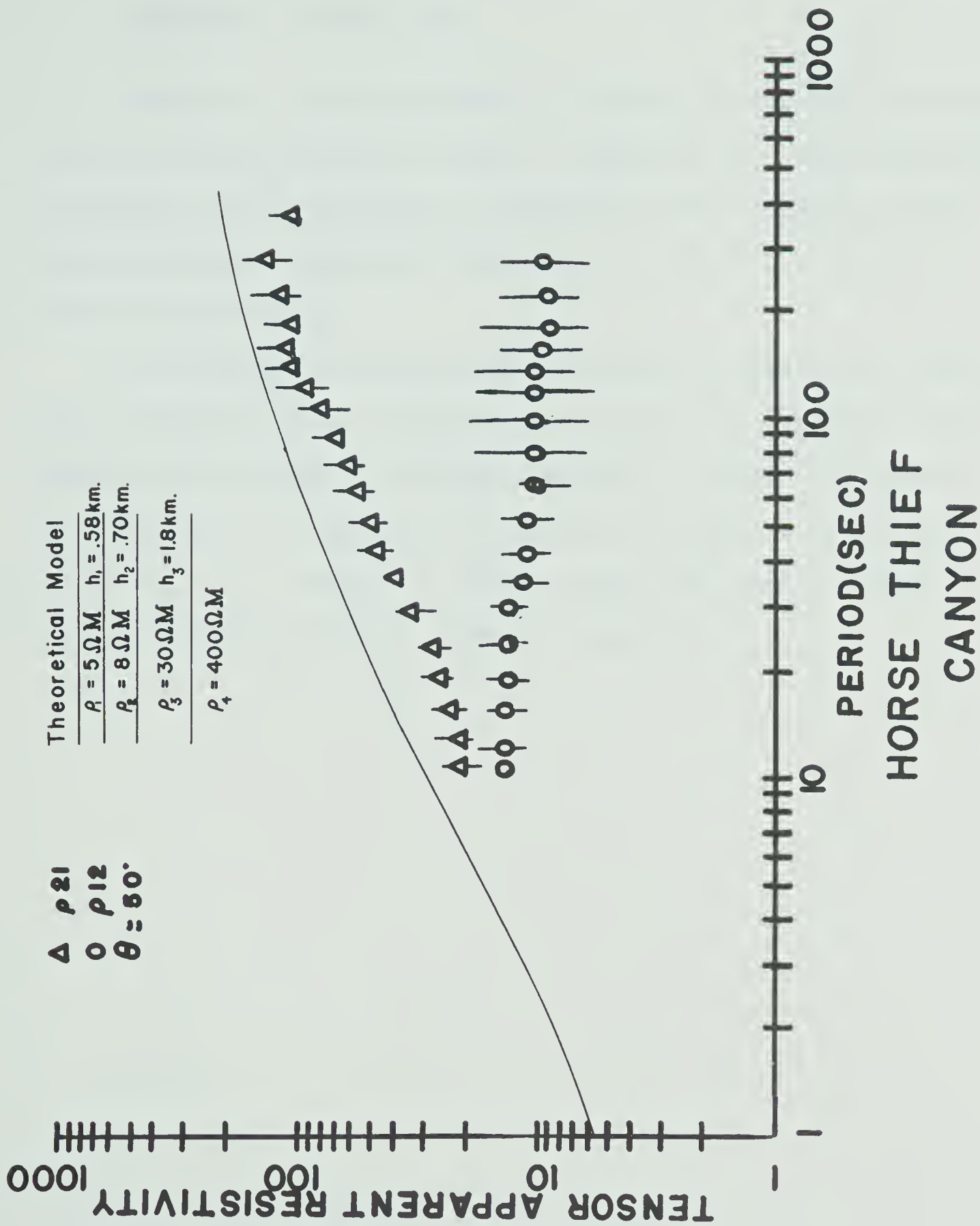
The relatively large scatter in the tensor ρ_{12} curve, Figure 4.2e, is caused by the magnetic signal being highly linearly polarized in the north-to-east direction with little power in the perpendicular direction.

*Figure 4.2d: Rotated diagonal tensor element for Horse
Thief Canyon location data at selected periods.*



HORSE THIEF
CANYON

Figure 4.2e: Tensor apparent resistivities for the Horse Thief Canyon location with a one dimensional model based on local well log information.



iii) Location 7 (L7) ($50^{\circ} 53'$ latitude, $112^{\circ} 35'$ longitude, elevation: +2800 feet)

Analysis of data recorded at Location 7 shows anisotropic resistivity properties as seen in Figure 4.2f. The principal directions of resistivity as defined by the rotation of the magnetotelluric impedance tensor as $\theta_1 = N 70^{\circ}E$ and $\theta_2 = N 30^{\circ}W$ (Figure 4.2g).

The topography around this location is essentially flat and no deviations in apparent resistivity are expected due to vertical variations in surface features. While no obvious features serve to explain the apparent anomalous splitting of the ρ_{12} , ρ_{21} curves it will be seen that both of these fit well on the profiles shown in Figure 4.4c.

Figure 4.2f: Tensor apparent resistivities for Location 7.

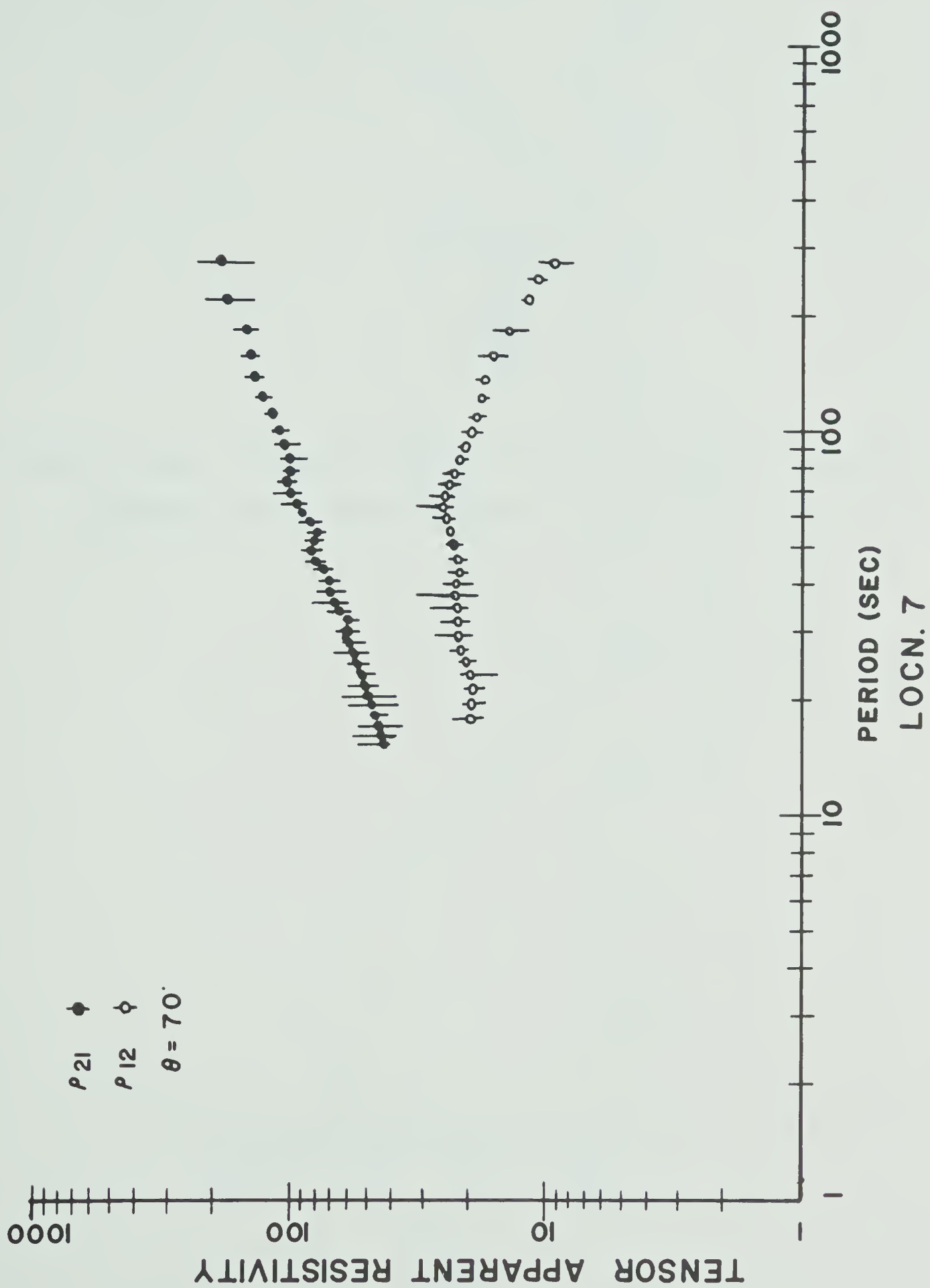
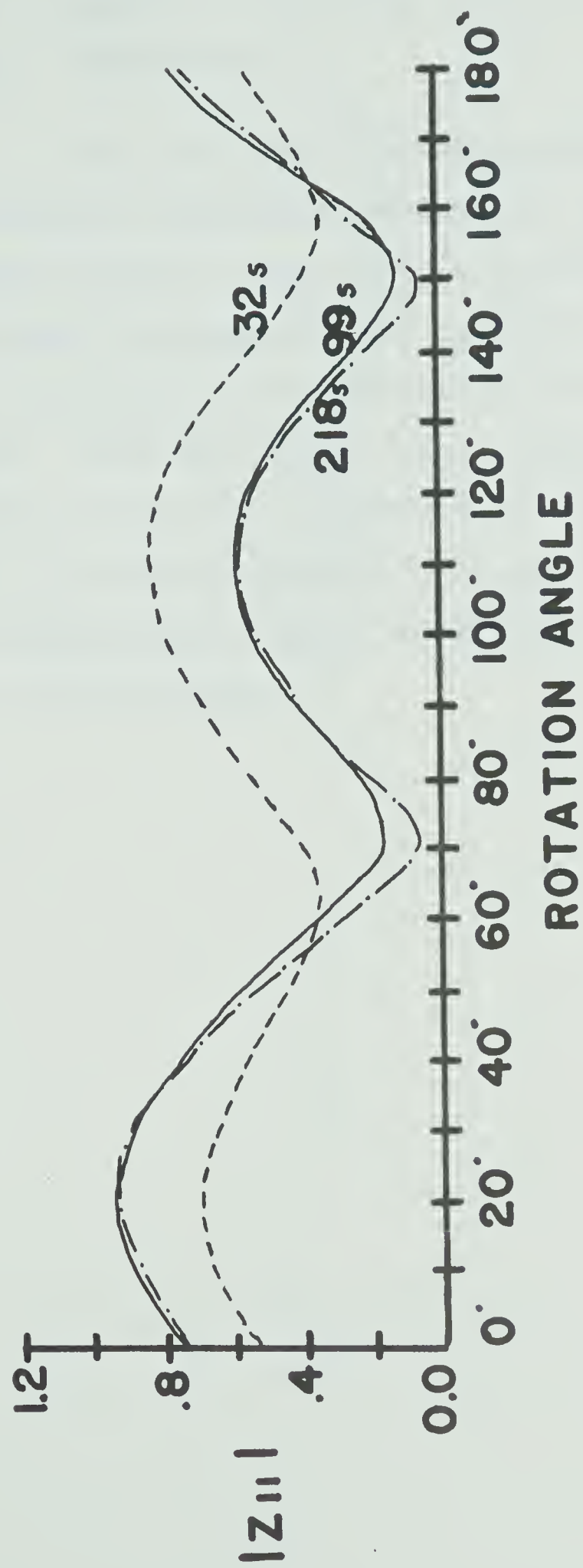


Figure 4.2g: Rotated diagonal tensor element as selected periods for Location 7 data.

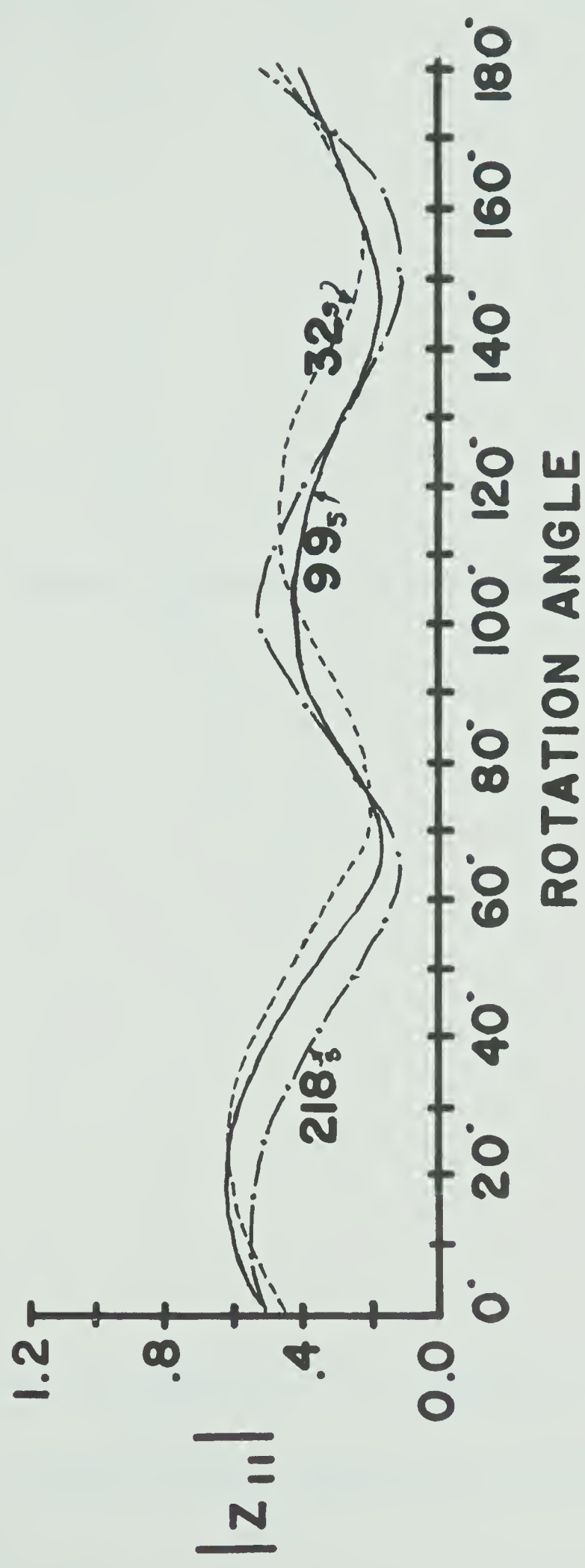


LOCN. 7

- iv) Location 5 (L5)(50° 34' latitude, 112° 30' longitude, elevation: +3000 feet)

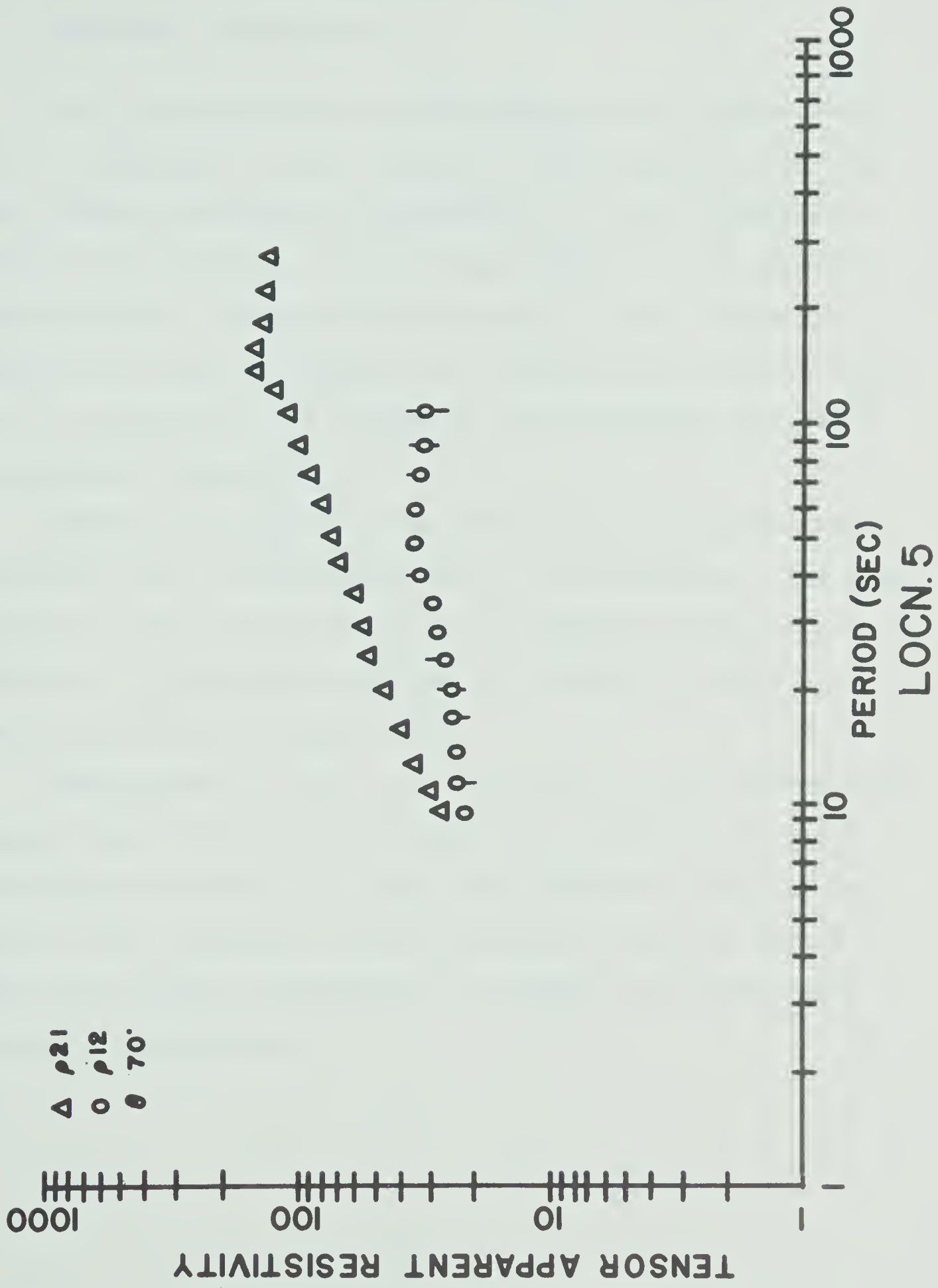
Location 5, like all other locations investigated in this study, exhibits anisotropic resistivity features. Regional surface relief in this area is virtually nil. Tensor analysis indicated two approximately orthogonal directions for the principal directions of resistivity given by $\theta_1 = N 70^\circ E$ and $\theta_2 = N 20^\circ - 30^\circ W$ (see Figure 4.2h). The rotated tensor ρ_{21} resistivity of Location 5 (Figure 4.2i) is very similar to the tensor ρ_{21} curve of Station 7. However the Location 5 ρ_{12} curve has appreciably greater values than ρ_{12} curve of Station 7 at the same period.

Figure 4.2h: Rotated diagonal tensor element at selected periods for Location 5 data.



LOCN. 5

Figure 4.2i: Tensor apparent resistivities for Location 5.



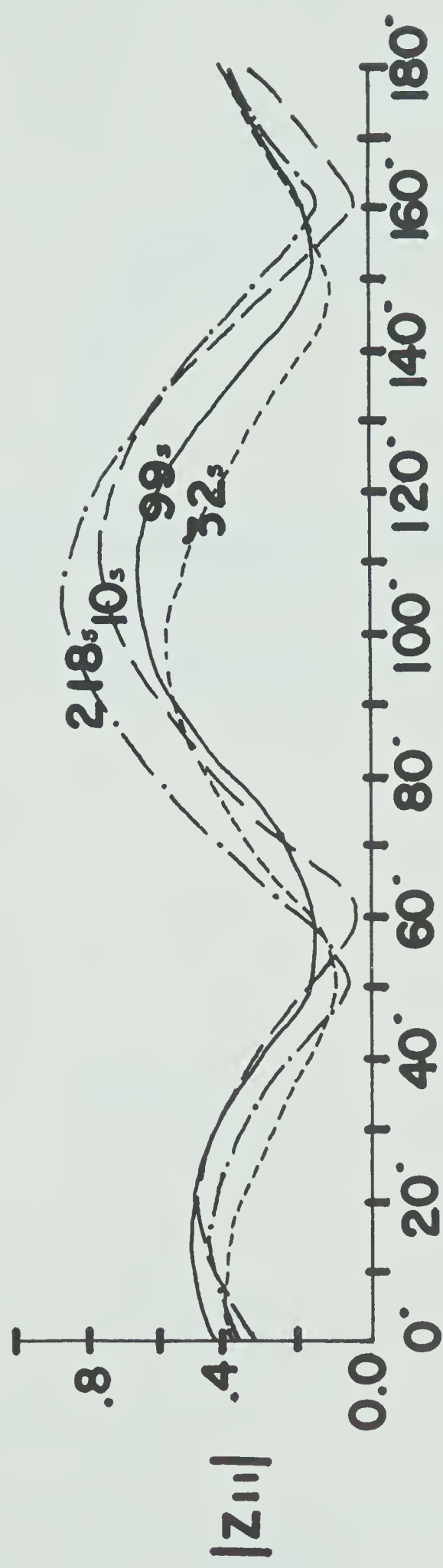
- v) Turin (TU) ($50^{\circ} 2'$ latitude, $112^{\circ} 31'$ longitude, elevation: +2700 feet)

This recording site is the southern most location which will be presented for the "eastern" magnetotelluric profile. Local surface features are reasonably flat and topographic effects are assumed to have no major effect on the apparent resistivities. Rotation of the diagonal tensor impedance elements as shown in Figure 4.2j, define the directions given by $\theta_1 = N 60^{\circ}E$ and $\theta_2 = N 30^{\circ}W$ as the approximate principal directions of resistivity.

Figure 4.2k shows the averaged polarization characteristics for two of the records used in the analysis. The high degree of linear polarization of the magnetic signal for periods $T > 100$ seconds explains the spread in sample $\rho_{1,2}$ resistivity values (Figure 4.2l).

Superimposed on the $\rho_{2,1}(T, \theta)$ curve is a one dimensional layered earth model interpretation of reasonably fit which was deduced from well log data. The basement lithology map, Figure 4.4b, indicates that this station lies in a region where the basement composition is assumed to be reasonably laterally homogeneous.

Figure 4.2j: Rotated diagonal tensor element at selected periods for Turin data.



ROTATION ANGLE

TURIN

Figure 4.2k: Instrument prewhitened magnetic power spectra and polarization characteristics (averaged over 6400 seconds) for a sample record recorded at Turin.

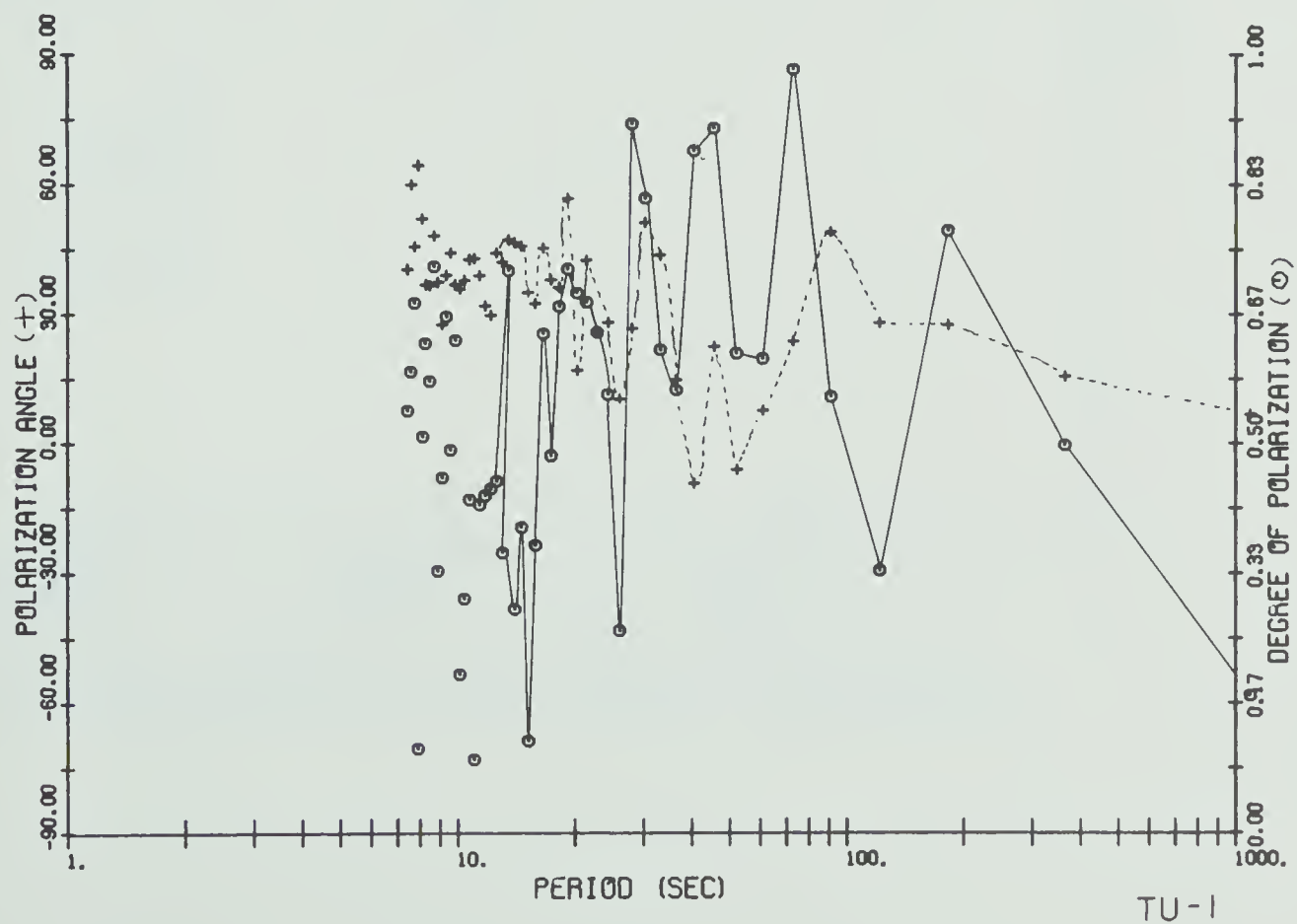
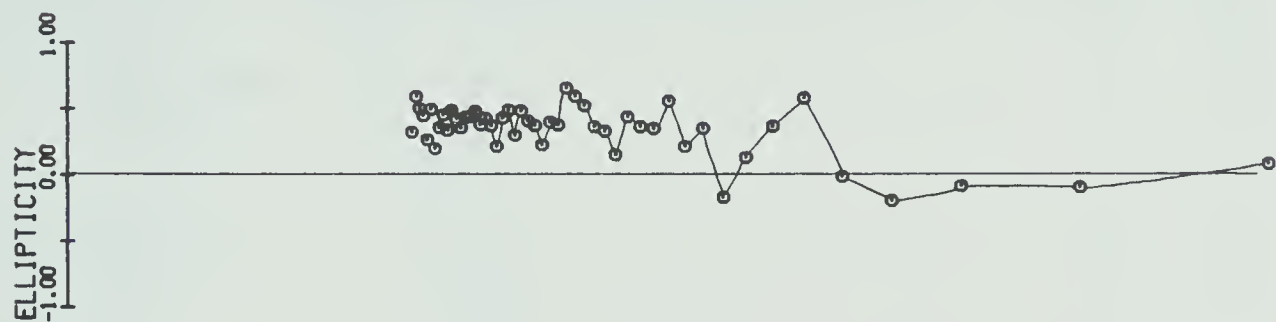
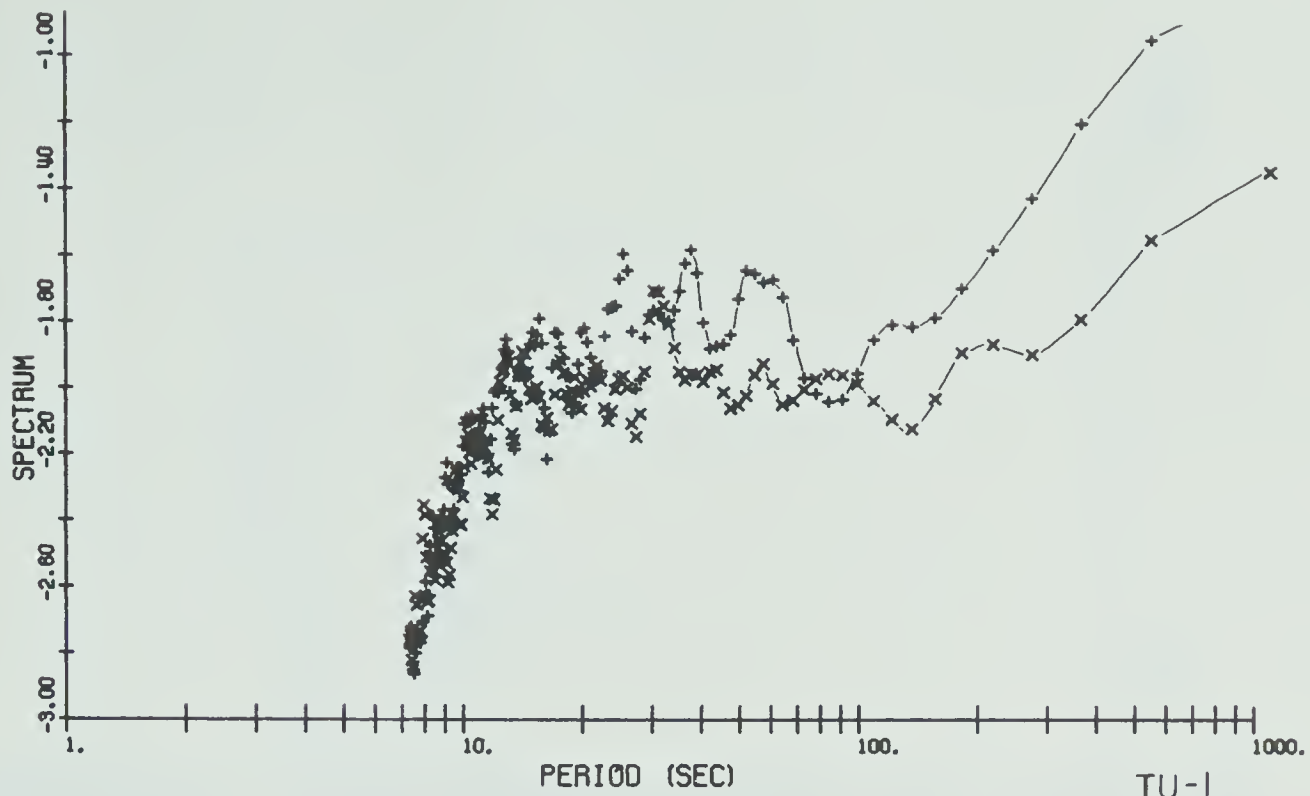
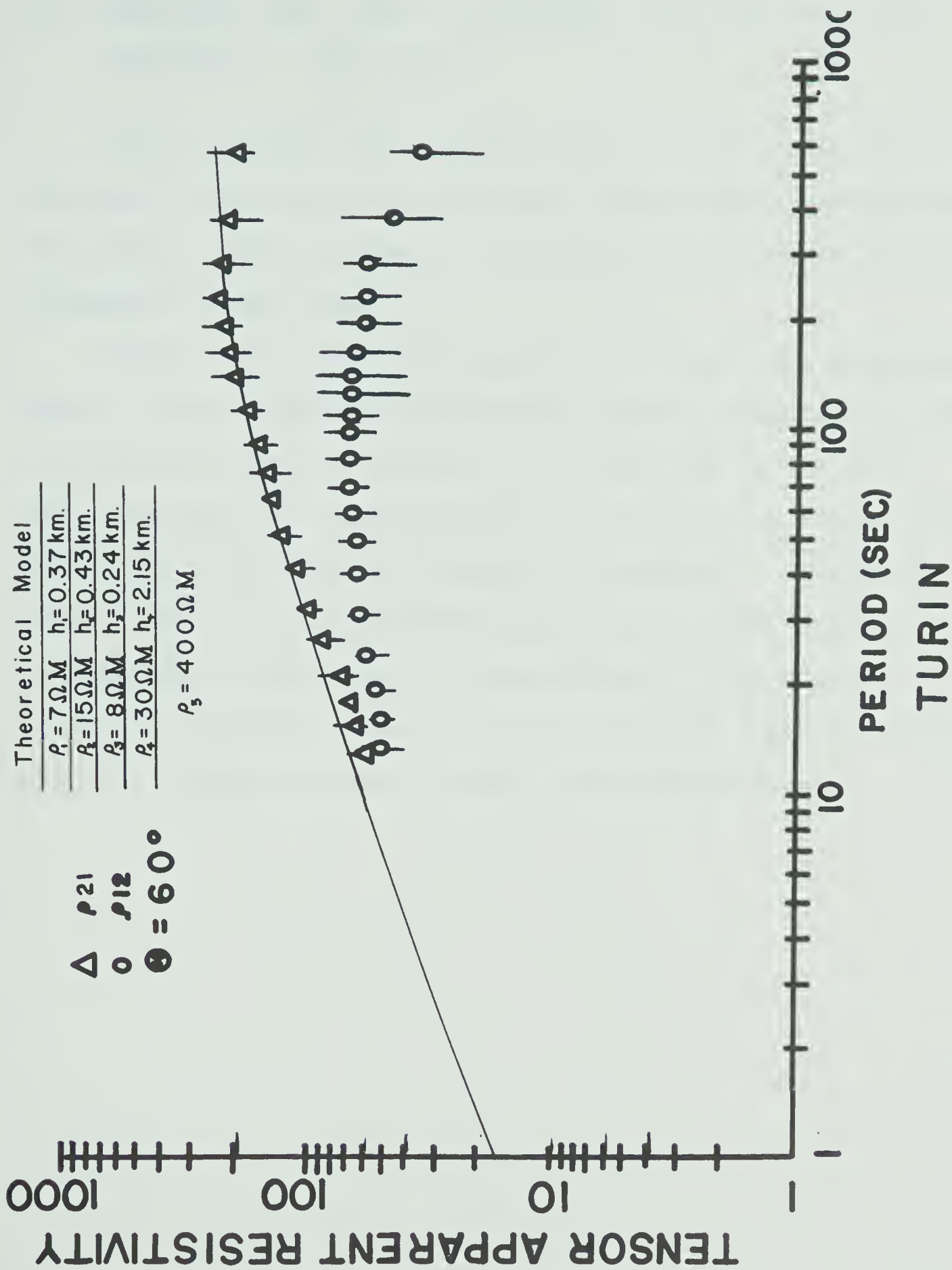


Figure 4.21: Tensor apparent resistivities for Turin location.

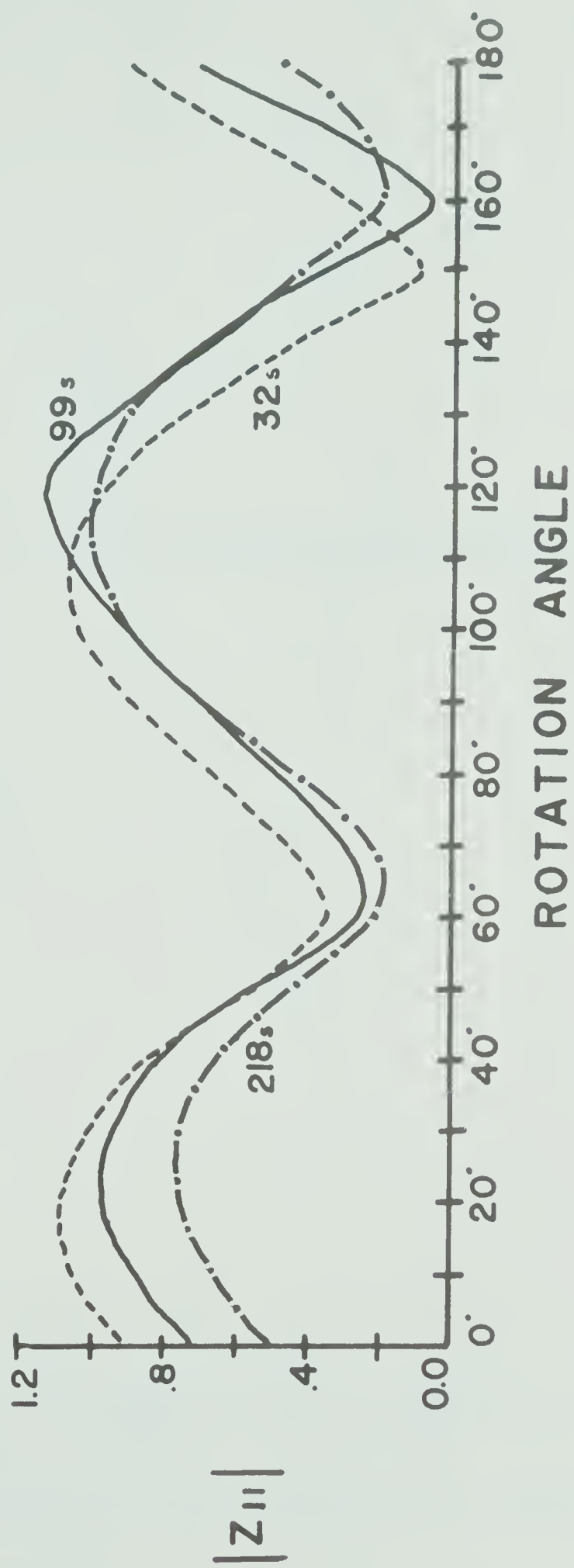


vi) Mossleigh (MO) ($50^{\circ} 48'$ latitude, $113^{\circ} 18'$ longitude, elevation: +3100 feet)

This recording site lies at the northern end of the "western" magnetotelluric profile. The location exhibits the same general characteristic anisotropy as all other stations presented in this work.

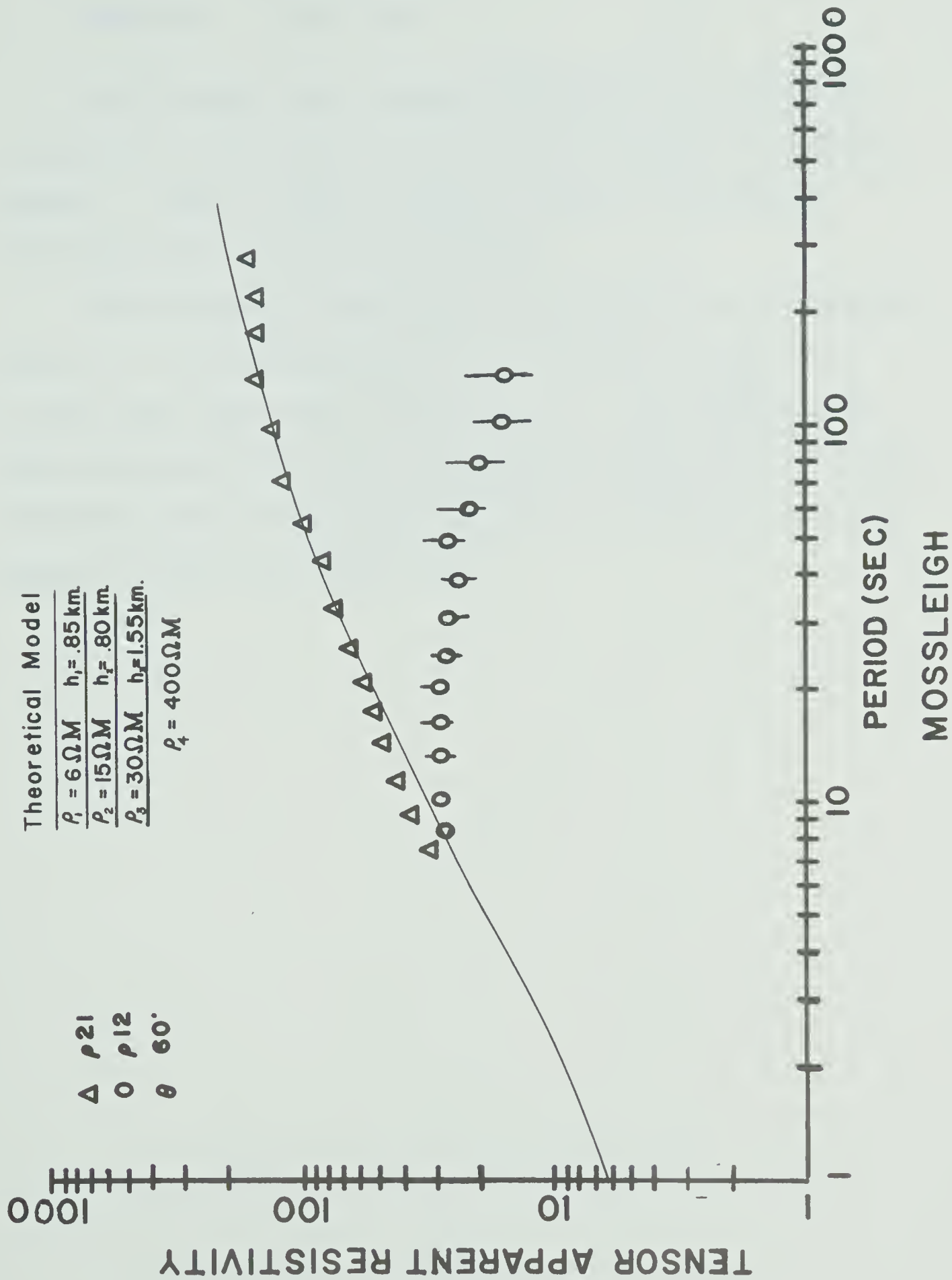
Figure 4.2m shows the results of rotating the impedance matrix. The minima of the diagonal tensor elements are shown to occur for rotation angles $\theta_1 = N 60^{\circ}E$ and $\theta_2 = N 30^{\circ}W$. The anisotropy in apparent resistivity, shown in Figure 4.2n, occurs for periods of ten seconds and greater. As seen in Figure 4.4b, at this recording site the basement composition is reasonably homogeneous. Superimposed on the tensor $\rho_{21}(T, \theta)$ curve is a model derived from local well log data, and is a reasonable match to the experimental curve.

Figure 4.2m: Rotated diagonal tensor element at selected periods for Mossleigh data.



MOSSLEIGH

Figure 4.2n: Tensor apparent resistivities for Mossleigh data.

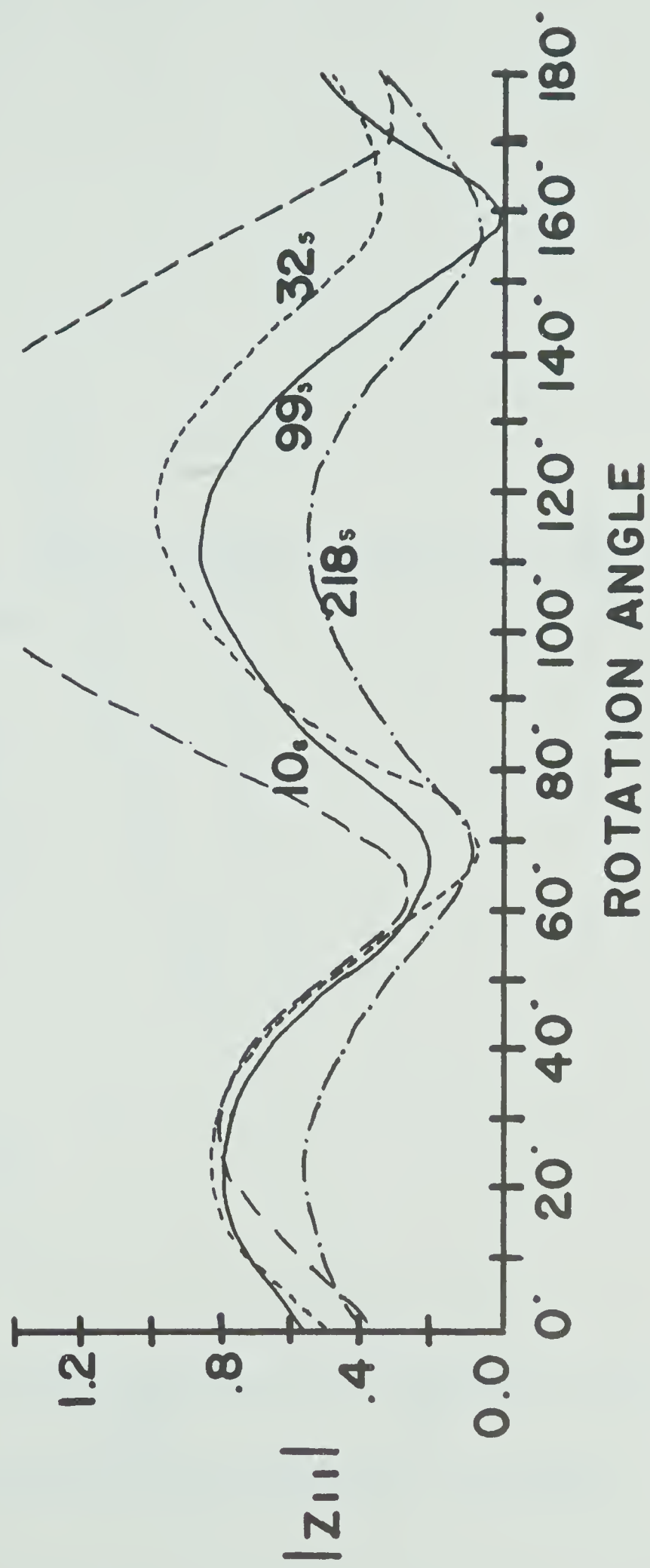


vii) Kilcardy (KC) ($50^{\circ} 17'$ latitude, $113^{\circ} 15'$ longitude, elevation: +3250 feet)

The Kilcardy and Carmangay locations are within 30 kilometers of the base of the foothill region in southern Alberta. Both of these stations exhibit a consistent anisotropy for all periods studied.

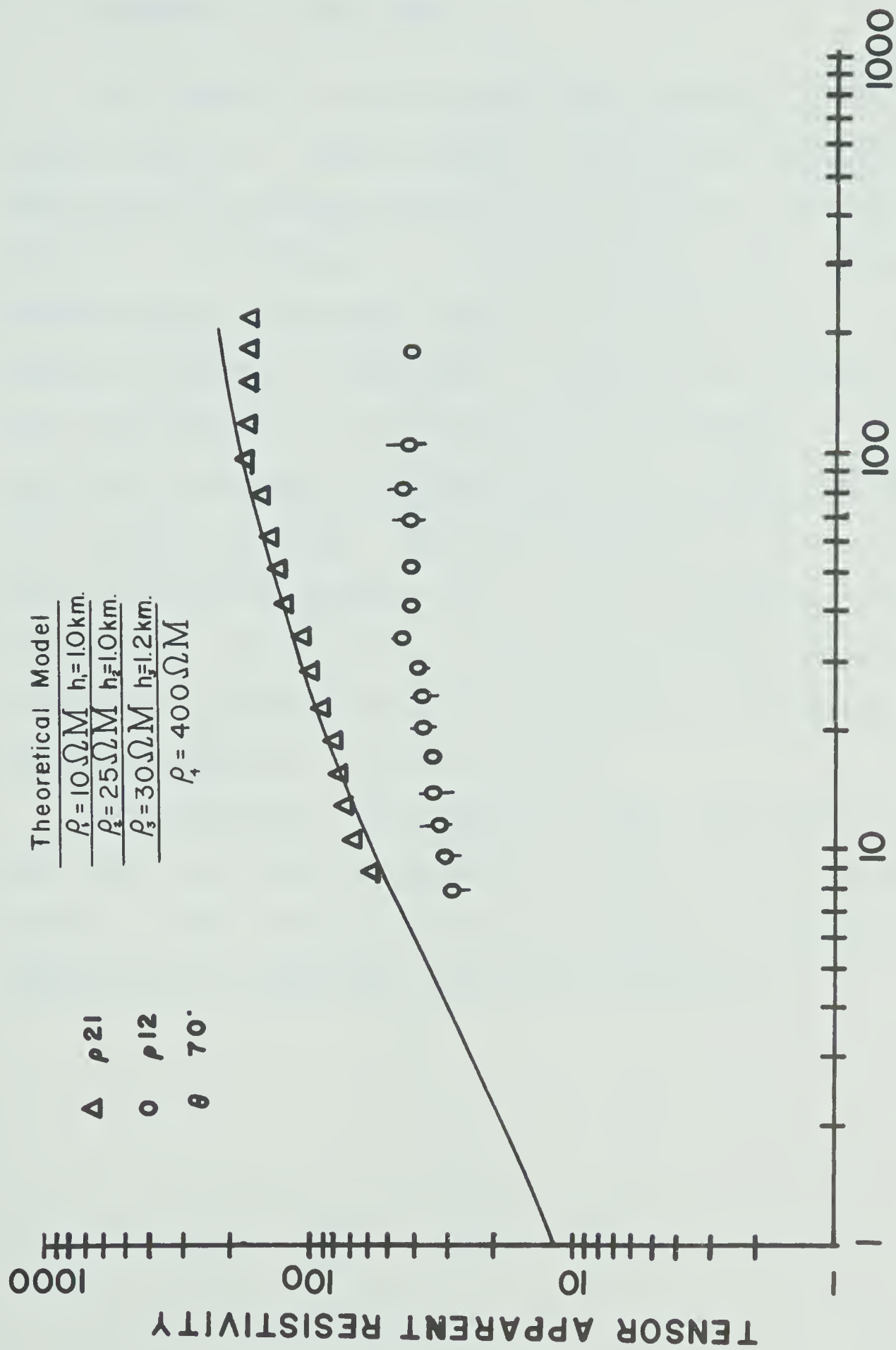
The principal directions of resistivity, as defined by $\min |z_{11}(T, \theta)|$, are defined by the rotation angles $\theta_1 \approx N 70^{\circ}E$ and $\theta_2 \approx N 20^{\circ}W$ (Figure 4.2o). A one dimensional model derived from local well log data appears to be a very reasonable fit to measured apparent resistivities for this station as seen in Figure 4.2p.

Figure 4.20: Rotated diagonal tensor element at selected periods for the Kilcardy data.



KILCARDY

Figure 4.2p: Tensor apparent resistivities for Kilcardy location.



Theoretical Model	
$\rho_1 = 10 \Omega M$	$h_1 = 1.0 km$
$\rho_2 = 25 \Omega M$	$h_2 = 1.0 km$
$\rho_3 = 30 \Omega M$	$h_3 = 1.2 km$
$\rho_4 = 400 \Omega M$	

Δ P_{21}
 $\circ$ P_{12}
 θ 70°

PERIOD (SEC)

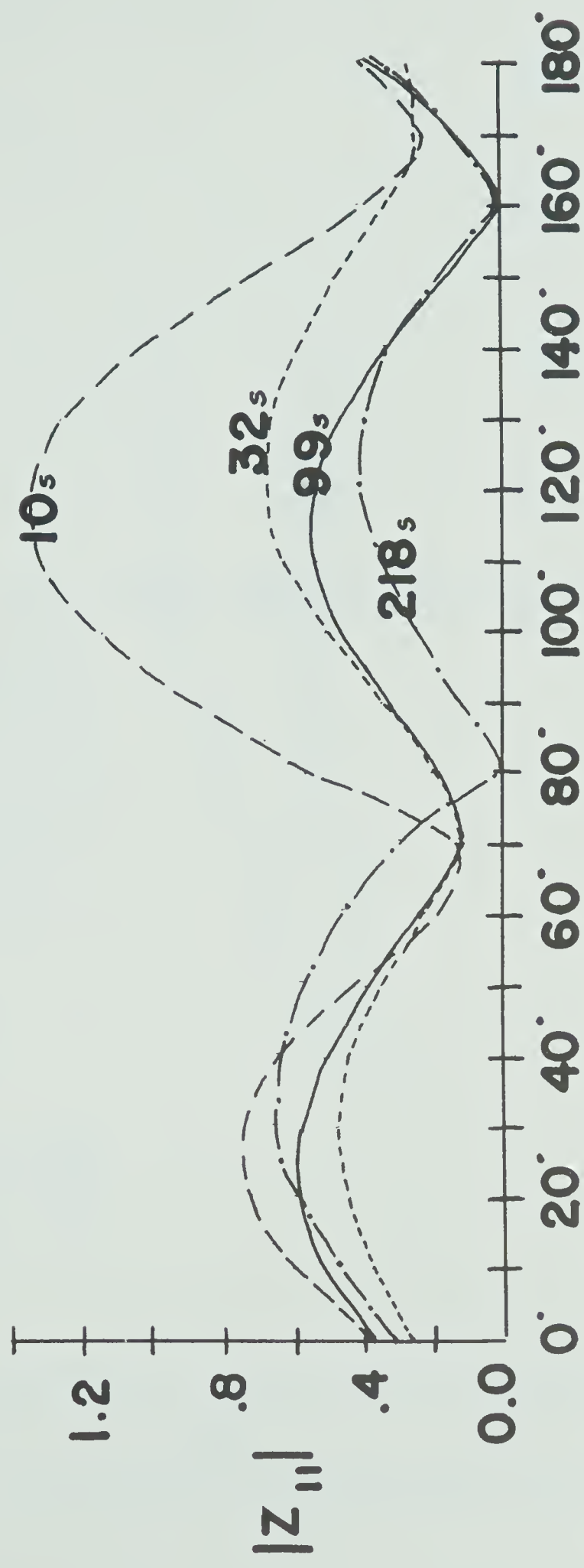
KILCARY

viii) Carmangay (CA) ($50^{\circ} 5'$ latitude, $113^{\circ} 18'$ longitude, elevation: +3250 feet)

This station is the southern most location of the western profile. Tensor analysis indicate two orthogonal directions of principal resistivity defined by the rotation angles $\theta_1 = N 70^{\circ}E$ and $\theta_2 = N 20^{\circ}W$ (Figure 4.2q). Agreement between scalar and tensor analysis was quite good. The low degree of scatter of the scalar resistivities indicated only very small changes in the polarization parameters. Figure 4.2r shows the tensor apparent resistivities obtained from data at this location. The dip observable on the $\rho_{21}(T, \theta)$ curve for periods greater than 100 seconds appears to be due primarily to the fall off in magnetic activity and the inability of spectral analysis techniques to reliably handle the steep slopes of the power spectra.

The theoretical one dimensional model was derived from local well log data and represents a good fit for shorter periods. This location is in an area where the basement composition is reasonably laterally homogeneous.

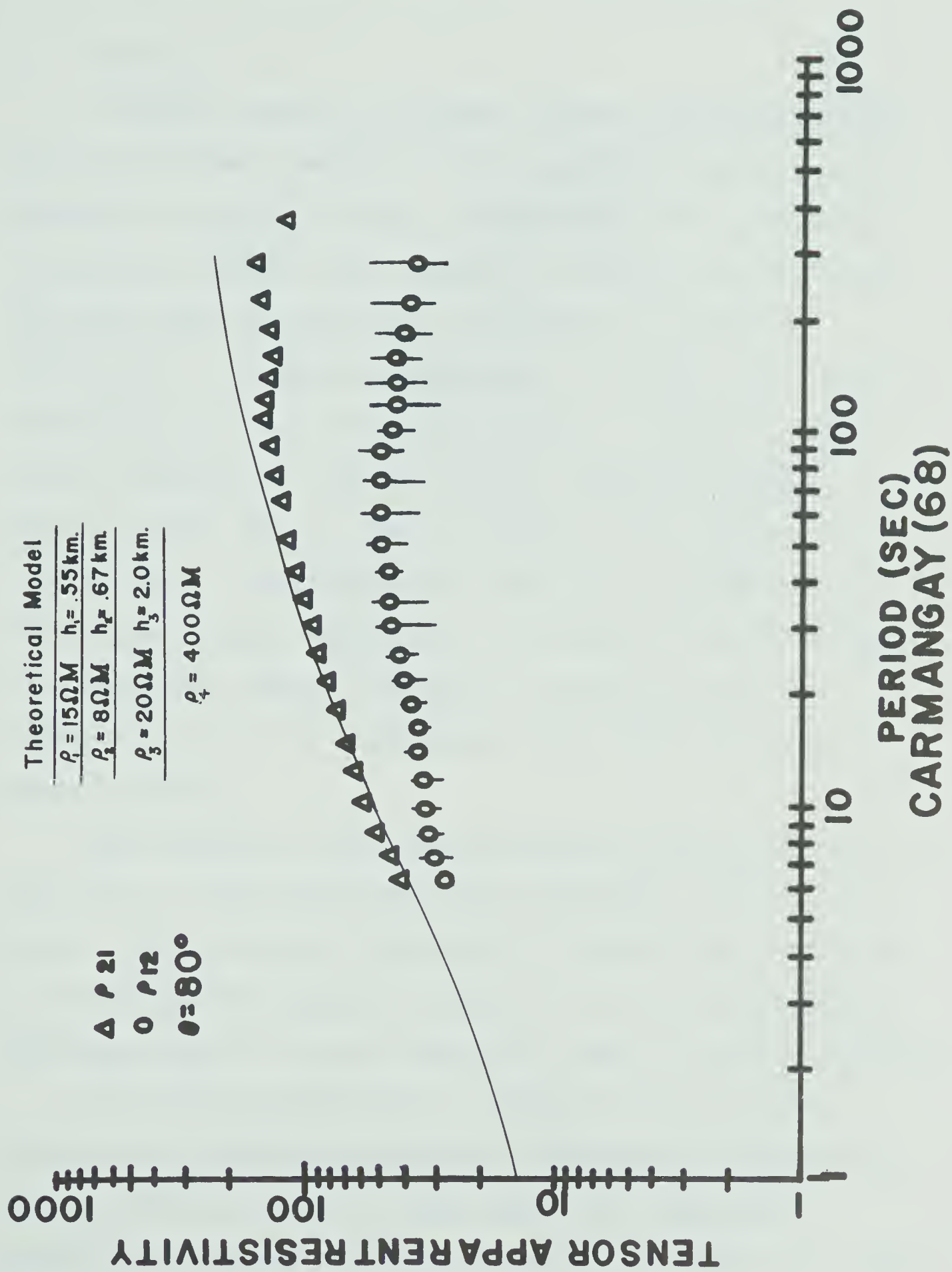
Figure 4.2q: Rotated diagonal tensor element at selected periods for Carmangay data.



ROTATION ANGLE

CARMANGAY

Figure 4.2r: Tensor apparent resistivities for Carmangay location.



4.3 Other Geophysical Studies in Southern Alberta

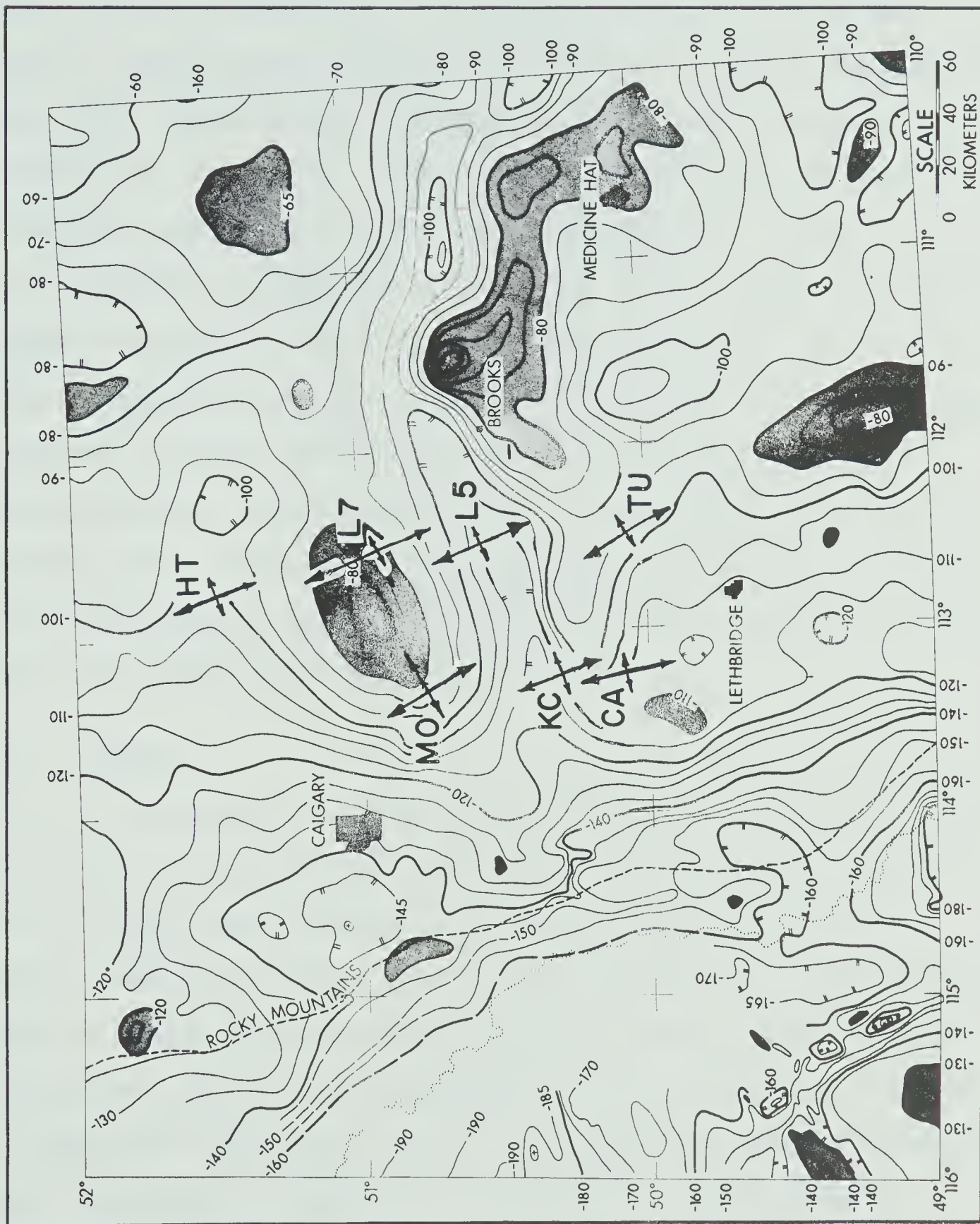
a) Gravity:

A gravity survey in southern Alberta has been carried out by the seismic members of the geophysics group at the University of Alberta using a Worden Model XPO gravimeter. The gravity readings were reduced to Bouguer gravity anomalies and combined with data provided by the Gravity Division of the Dominion Observatory of Canada, Gulf Canada Limited, and Chevron Standard Limited. A total of 1800 gravity readings was reduced and the complete Bouguer gravity anomaly map is shown in Figure 4.3a (Kanasewich et al - 1969). The pronounced east-west trending Bouguer gravity low extends across most of southern Alberta except for a positive anomaly northeast of Brooks called the Princess high and the north-south trend over the Rocky Mountain System.

Superimposed on the Bouguer gravity anomaly map are the locations at which magnetotelluric recording stations were placed. The principal directions of resistivity, obtained by minimizing the diagonal elements of the magnetotelluric impedance tensor, are also shown for seven recording stations.

The overall correlation of Bouguer gravity anomaly contours and principal resistivity directions is very good, implying that at least to some degree the subsurface geophysical features are being detected and trends are being outlined by the magnetotelluric technique.

Figure 4.3a: Bouguer gravity anomalies in southern Alberta (after Kanasewich et al - 1969) and the principal directions of resistivity for seven magnetotelluric recording locations. The larger apparent resistivities are indicated as the axis of increased length.



b) Magnetometer Survey:

A ground magnetometer survey in the southern Alberta area was also compiled by the seismic members of the geophysics group at the University of Alberta using a Barringer Model GM - 102A nuclear precession magnetometer with an accuracy of 10 gammas. The residual total magnetic field intensity map, Figure 4.3b, (after Kanasewich - 1968) was computed by subtracting the first seven harmonics of the regional magnetic field from the observed magnetic values. The 100 gamma contours on the residual magnetic intensity map displays a strong east-west trend in the region of the magnetotelluric recording stations. Superimposed on the residual map contours are the principal directions of resistivity obtained by the magnetotelluric tensor analysis of data from each magnetotelluric recording station.

c) Seismic:

On the basis of reflection seismology in this region and the gravity and magnetic survey previously mentioned, Clowes et al (1968) have produced the generalized crustal cross section shown in Figure 4.3c. The fault shown in the central part of the section has been confirmed seismically, but the postulated fault on the southern end of the profile is based on ambiguous reflection data. The gravity calculations show that the fault is required to produce the steep gravity gradient. (Clowes - 1969).

Figure 4.3b: A residual total magnetic field intensity map for southern Alberta (after Kanasewich - 1968) with the principal directions of resistivity for seven magnetotelluric recording locations. The larger apparent resistivities are indicated as the axis of increased length.

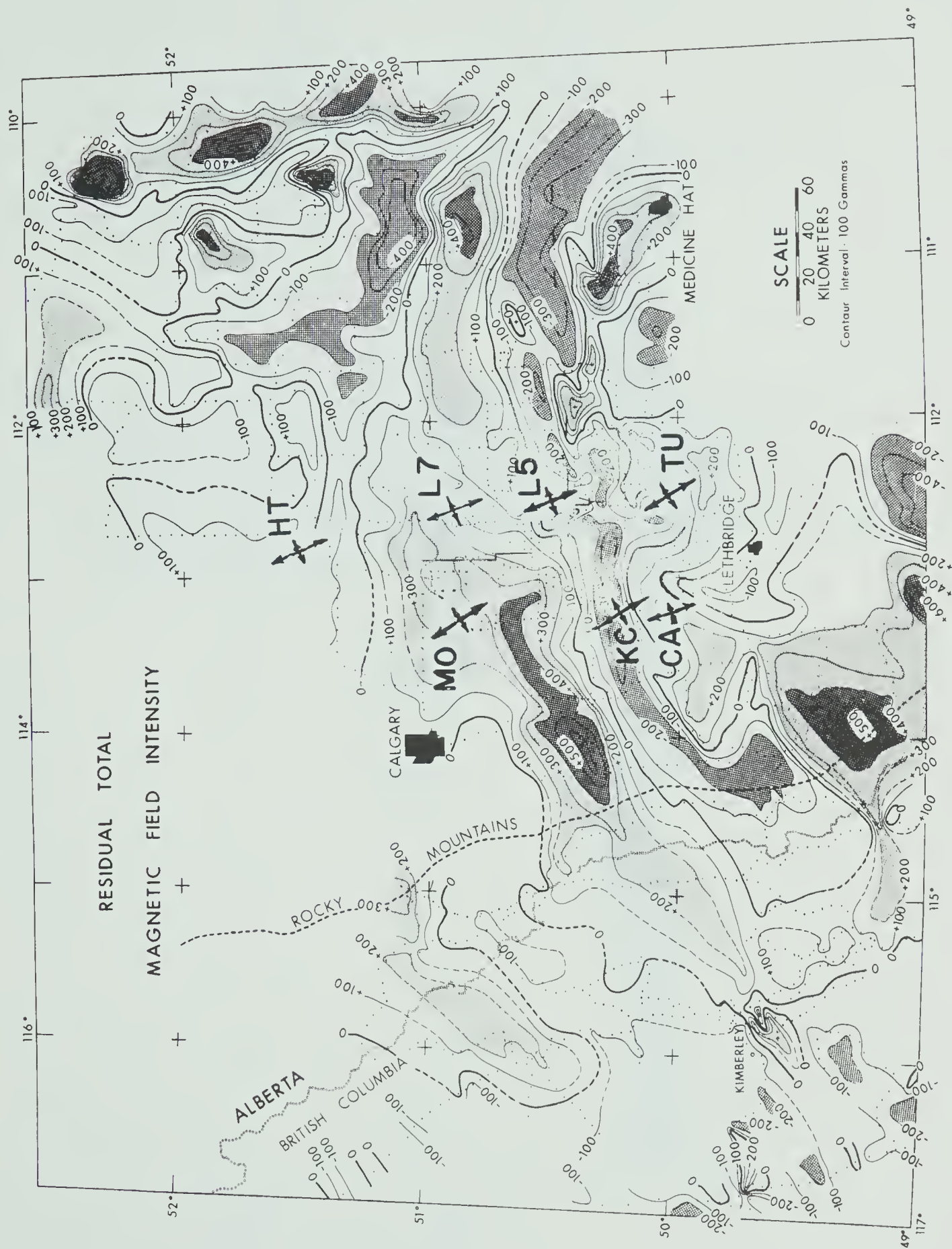
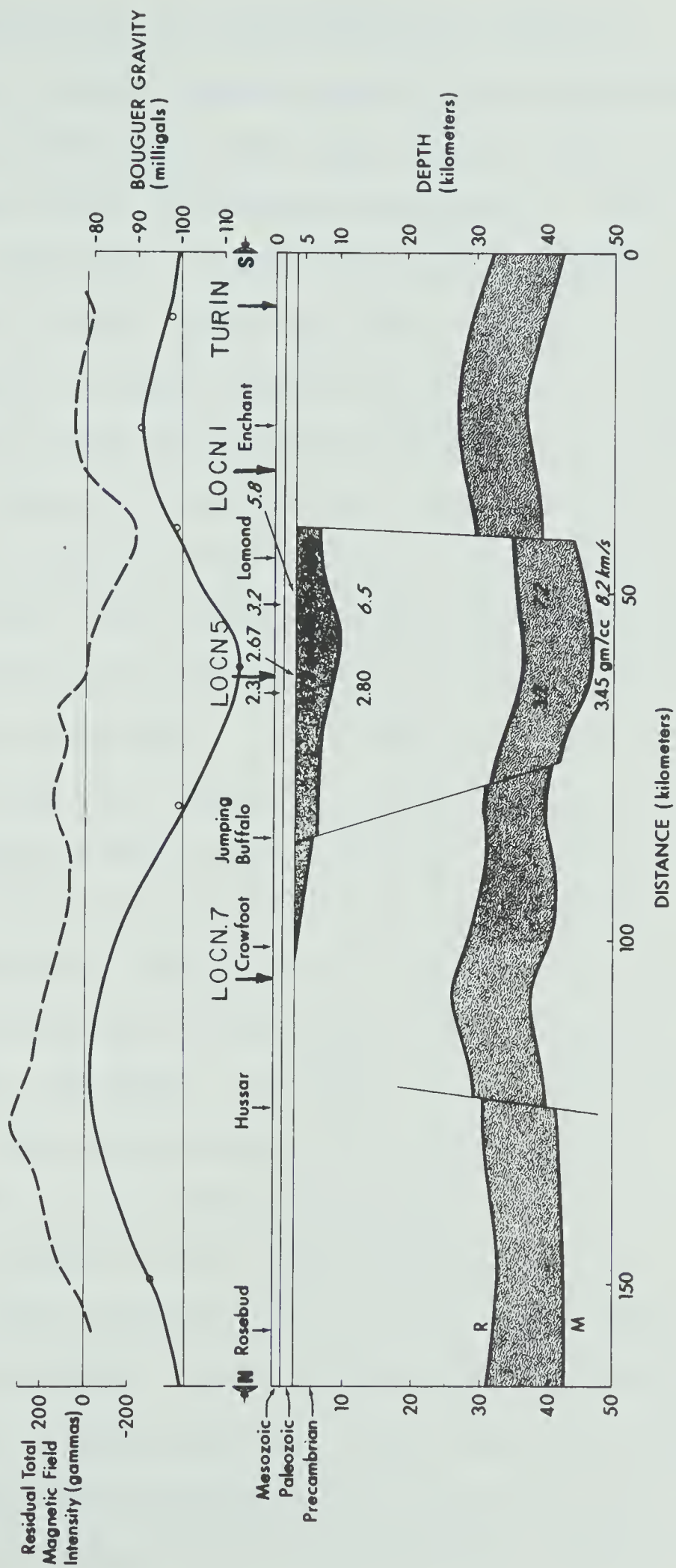


Figure 4.3c: Generalized north-south cross section of residual total magnetic field intensity, Bouguer gravity, and generalized seismic cross section for southern Alberta.



DEPT. OF AGRICULTURE

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4.4 Interpretation of Magnetotelluric Results.

The ρ_{21} tensor apparent resistivity curves shown in section 4.2 generally agree with the work done by others in the Alberta plains (Srivastava and Jacobs - 1963; Vozoff, Hasegawa, and Ellis - 1963; Srivastava, Douglass, and Ward - 1963; Ellis - 1964; Vozoff and Ellis - 1966). However there is only a slight resemblance of the ρ_{12} apparent resistivity curves with those published by the above mentioned workers. This is partially due to the fact that previously only scalar resistivities have been calculated and this work represents the first regional magnetotelluric survey in Alberta using tensor analysis and rotation into the principal resistivity directions. I. K. Reddy (private communication) has found that data from other magnetotelluric recording sites in the Alberta plains which have been analysed using tensor techniques also show similar anisotropic behavior.

To interpret accurately the resistivity curves at this time would certainly be desirable but not enough information is presently available. The lack of high resolution data in the 100 - 1000 second range is a great drawback. Several workers (Niblett and Sayn-Wittenstein - 1960; Srivastava et al - 1963; Vozoff et al - 1963; Vozoff and Ellis - 1966) have presented magnetotelluric results from Alberta which reportedly indicate a high conductivity zone between 60 - 150 kilometers. If this zone does exist then the apparent resistivity curves presented here will not give a clear indication of deep crustal resistivities (which could be of

the order of $10^3 - 10^4 \Omega\text{M.}$) due to limited bandwidth of the recording apparatus. If the high conducting zone does not exist then the deep crustal resistivities must be approximately $300 - 400 \Omega\text{M.}$ which does agree with values from well log data which penetrate the Precambrian surface in Alberta.

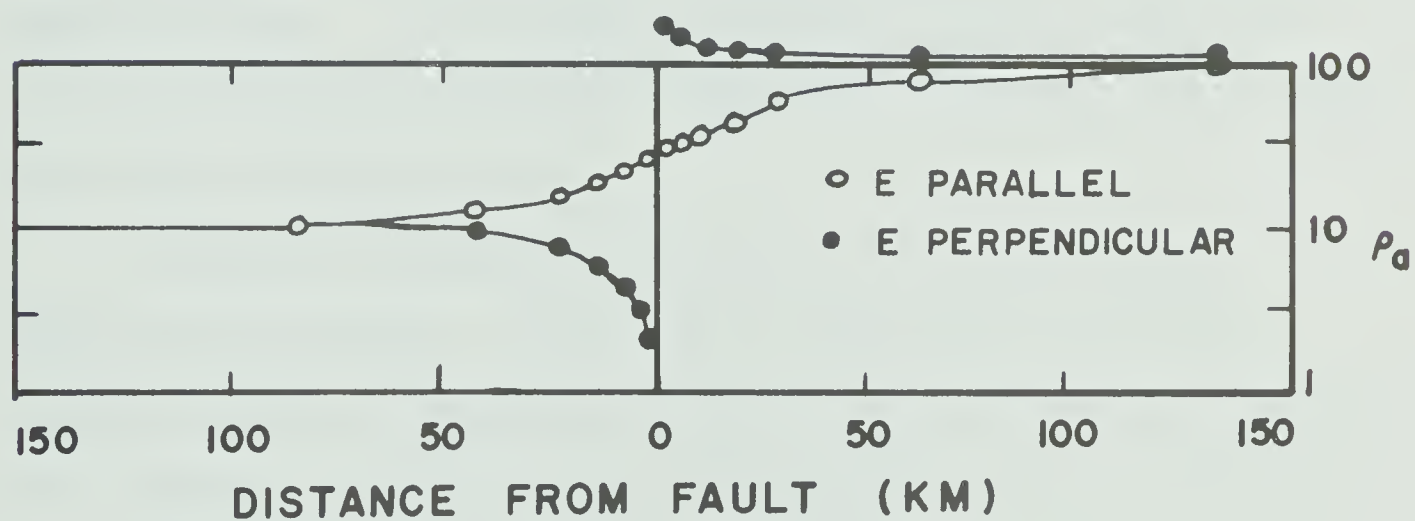
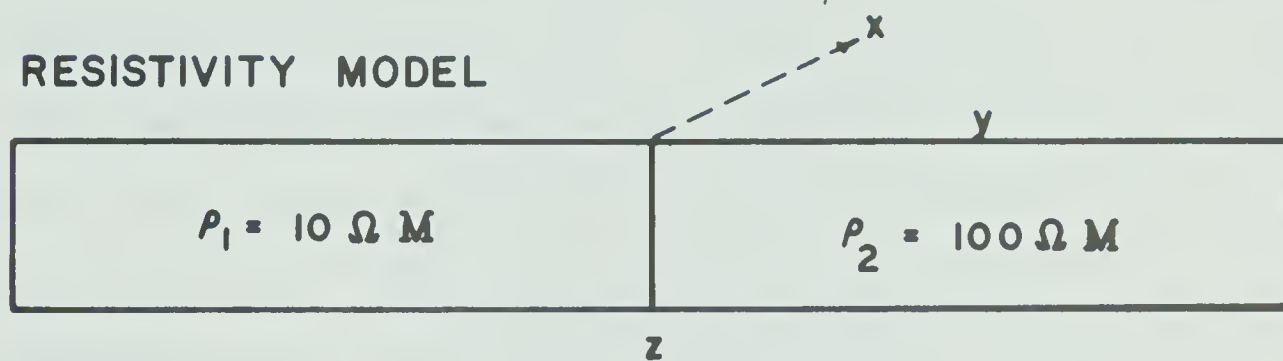
Caner and Auld (1968) have presented scalar resistivity curves taken at Victoria, B.C. These curves show apparent resistivities of $5000 \Omega\text{M.}$ for $T \approx 3$ seconds which decrease to $2000 \Omega\text{M.}$ for $T = 80$ seconds. This indicates the Rocky Mountain/Sedimentary Basin System, to a first order approximation, might be treated as a vertical fault with high resistivity values to one side and low resistivity values on the opposite side of the fault.

The case of a vertical contact fault with an infinitely deep basement has been treated analytically by d'Erceville and Kunetz (1962; see section 2.3). In this model the electric field is normal to the fault strike. Figure 2.3a (after d'Erceville and Kunetz) shows the behavior of the $|\frac{E}{H}|$ ratio as the fault is approached for various resistivity contrasts. As the fault is approached from the low resistivity side, the $|\frac{E}{H}|$ ratio drops rapidly in value. The ratio is discontinuous at the fault boundary, and then assumes a large value which converges to the value of $|\frac{E}{H}|$ for the non-fault case as $x \rightarrow \infty$. Swift (1968) has used a network solution approximation to calculate theoretical apparent resistivities for a vertical contact fault ($\rho_1 = 10 \Omega\text{M.}$, $\rho_2 = 100 \Omega\text{M.}$, period = 10^3 seconds) for the electric

Figure 4.4a: Theoretical field relationships over an infinite vertical contact fault (after Swift - 1967).



RESISTIVITY MODEL



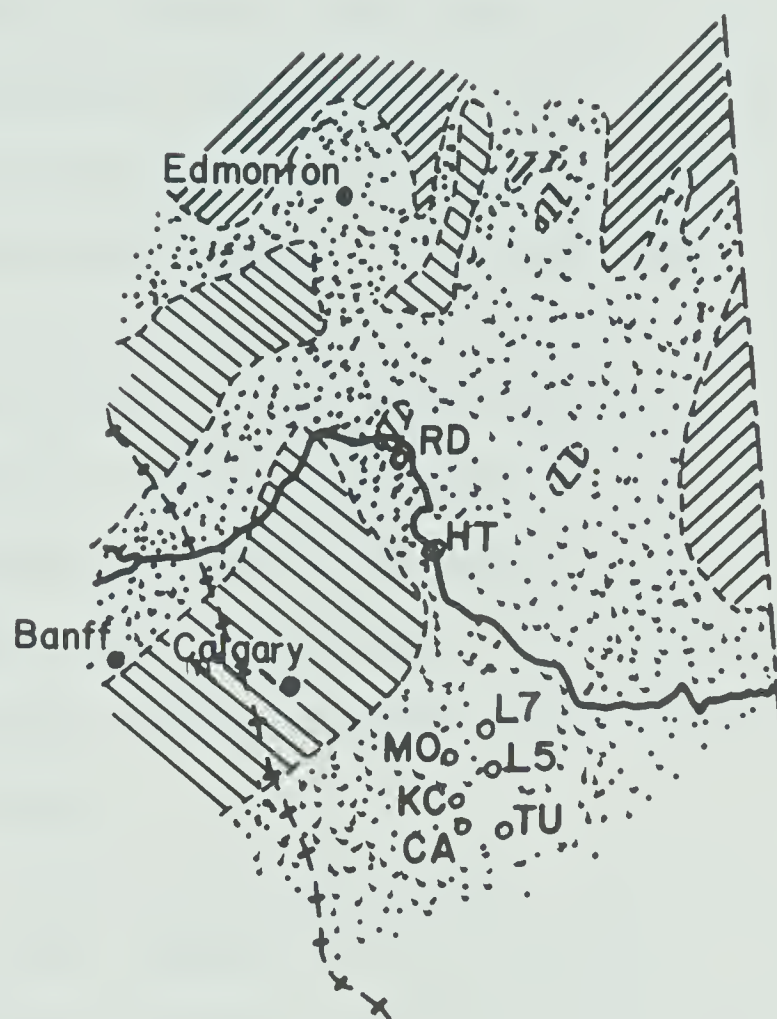
PERIOD = 10^3 SEC.

polarization parallel and normal to the vertical contact (see Figure 4.4a).

This situation appears to model the results obtained in the sedimentary basin in Alberta (the low resistivity side) in this work. The $\rho_{12}(T, \theta)$ curves (corresponding to the $E_{\perp}$ case) have much lower values than the $\rho_{21}(T, \theta)$ values (corresponding to the $E_{\parallel}$ case). Since there are still significant deviations of theoretical apparent resistivity curves for the $E_{\perp}$ case with 10:1 resistivity ratio at 100 kilometers from the fault (and since all magnetotelluric locations presented here are within 100 kilometers of the base of the foothills of the Rocky Mountain System) it is doubtful whether the ρ_{21} curves give a true assessment of subsurface resistivities. It is highly probable that the apparent resistivities presented here are lower than the true values of crustal resistivities.

A second interpretation possibility also exists. Garland and Burwash (1959) have stated that the Bouguer anomaly field over central Alberta must be due to lithological changes in the basement which trend to the northeast in the sedimentary basin. The lithology of the Precambrian basement as inferred from well samples and gravity anomalies is shown in Figure 4.4b. Their rough calculations indicate that the changes in lithology begin somewhere near the Precambrian surface and could extend as deep as nine kilometers into the basement. Rankin and Reddy (1969) have attempted to interpret scalar resistivity curves from the

*Figure 4.4b: Lithology of the Precambrian basement as
inferred from well samples and gravity anomalies
(after Garland and Burwash - 1959).*



MT RECORDING LOCATION ○
 FOOTHILLS FRONT (SURFACE) +
 BASEMENT LITHOLOGY

	Chiefly Gneiss
	Granitic
	Basic Phases

University of Alberta Geophysical Observatory, Edmonton, Alberta, in terms of the subsurface features. Their model indicates an anisotropic layer of 10:1 resistivity contrast from 5.5 kilometers to 14.5 kilometers in depth.

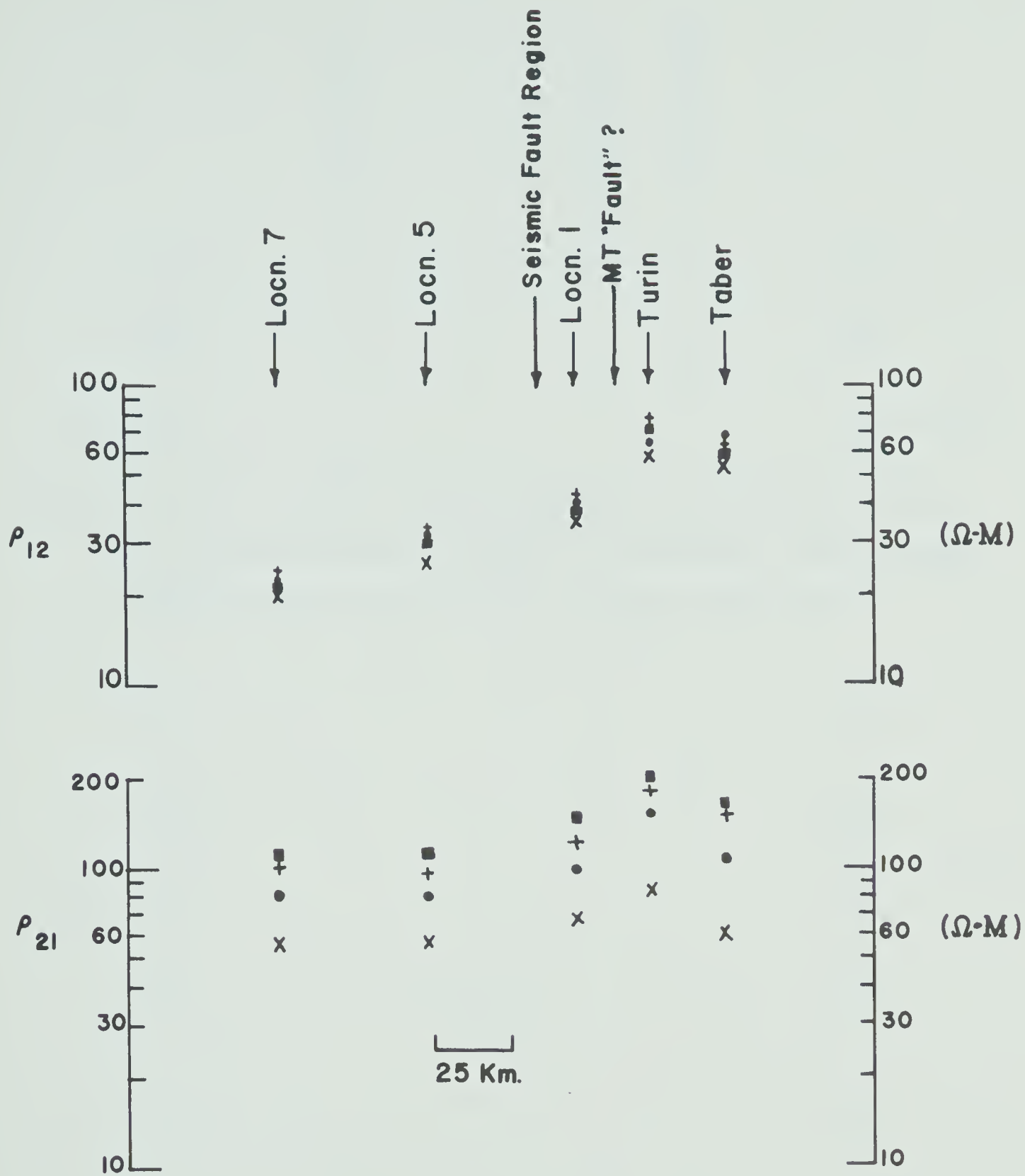
It should be noticed also that the closer the magnetotelluric recording location was to the base of the foothills, the shorter the period at which the anisotropy in apparent resistivity begins. Thus the Red Deer apparent resistivity curves, roughly 80 kilometers from the base of the foothills exhibit a divergence beginning at 20 seconds, while the closest stations to the foothills, Carmangay and Kilcardy, exhibit anisotropies which occur at periods considerably less than ten seconds. For these reasons, it is felt that lateral inhomogeneities must induce the major part of the anisotropy.

Despite the overall anisotropic character of apparent resistivity which appears to be consistent over a large area, it still appears possible to garner some general information about local subsurface structure by magnetotelluric methods. Figure 4.4c shows the apparent resistivity from the "eastern" north-south magnetotelluric profile plotted for four periods (25, 50, 75, and 100 seconds). The approximate location of the southern seismic fault is also shown. If the apparent resistivities do indicate a vertical fault it would appear to lie between stations Turin and Location 1. Figure 4.4d shows the same type of diagram for the "western" profile,

which while consistent with the eastern profile does not have enough coverage to be interpreted at present.

Figure 4.4c: Rotated tensor apparent resistivity values for selected periods along the "eastern" north-south magnetotelluric profile in southern Alberta.





PERIOD
 x 25 sec
 • 50 "
 + 75 "
 ■ 100 "

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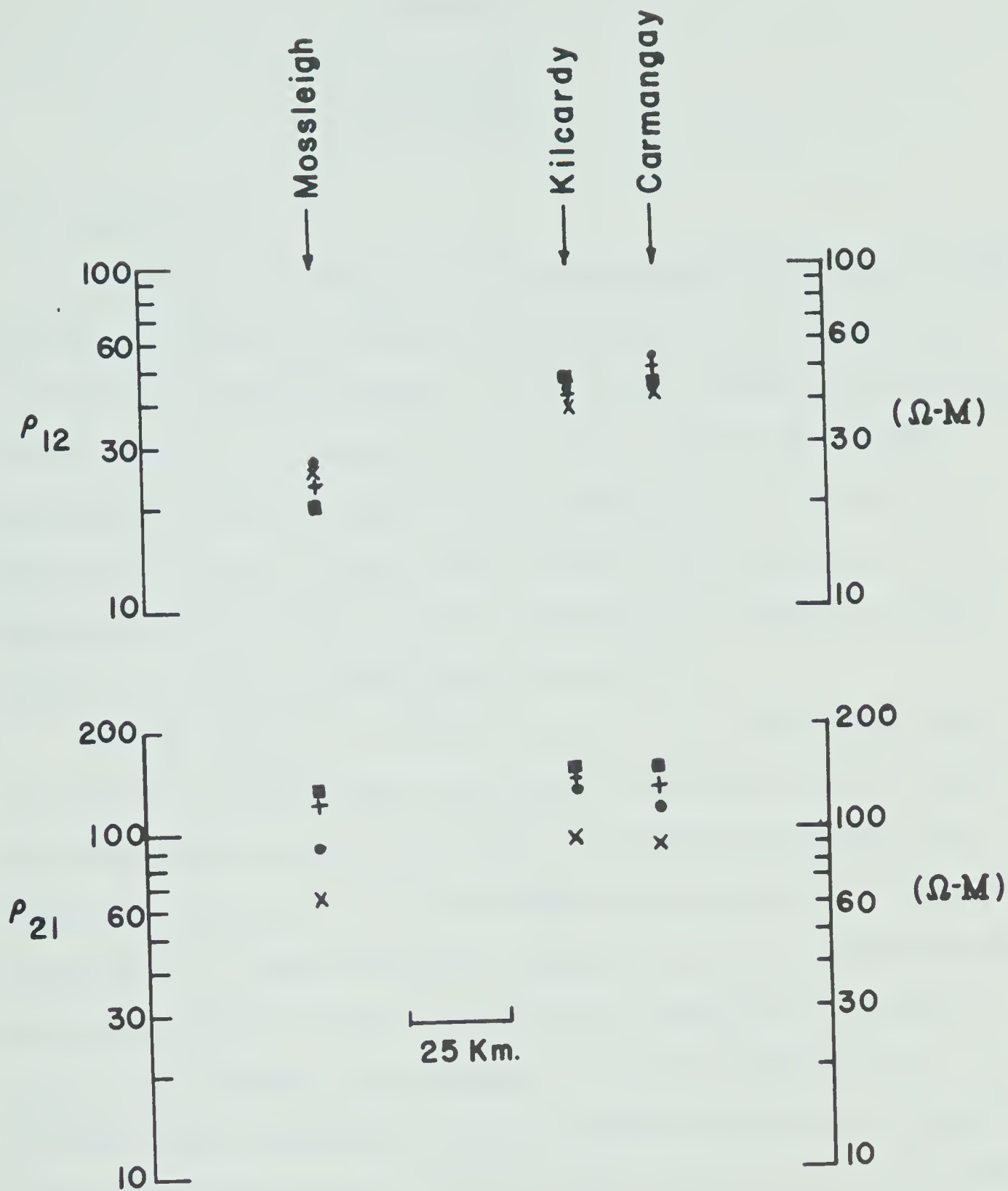
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*Figure 4.4d: Rotated tensor apparent resistivity values
for selected periods along the "western" north-south
magnetotelluric profile in southern Alberta.*



PERIOD

x 25 sec

• 50 "

+ 75 "

■ 100 "

CHAPTER V

CONCLUSIONS

5.1 Conclusions

Techniques of analysis have been presented which produce consistent apparent resistivity estimates for periods of 10 to several hundred seconds. It has been indicated that some of the scatter in apparent resistivity curves previously presented by other workers using techniques of spectral analysis is due to steep slopes in the power spectra of the magnetotelluric fields and the inability of spectral techniques to reliably handle this situation.

Tensor analysis of magnetotelluric data has been shown to indicate a consistent and marked anisotropy for all magnetotelluric recording locations presented in central and southern Alberta. It has also been shown that the principal directions of tensor apparent resistivity show a correlation with both Bouguer gravity anomaly and residual total magnetic field trends. An argument has been presented to indicate that the Rocky Mountain System represents a lateral resistivity inhomogeneity which would appear as an anisotropy in apparent resistivity throughout the sedimentary basin. Modeling the Rocky Mountain/Sedimentary Basin as a vertical fault of 10:1 resistivity contrast offers tentative explanation for the observed anisotropies in apparent resistivity. This very crude model is greatly over simplified but

constitutes a major step towards understanding the true resistivity structure in Alberta.

Subsurface structure in central and southern Alberta generally trends in the northeast direction and these structures may induce a portion of the anisotropy. An accurate evaluation however of the relative magnitude of this effect is not possible with the present level of knowledge and degree of sophistication of interpretational techniques.

It has also been shown that magnetotelluric profiling the region of seismically located precambrian rift valley faults produced evidence which can be interpreted as a vertical contact resistivity fault and can be used to interpret local structural effects.

5.2 Suggestions for Future Work

The results obtained in this work indicate that an extensive program of magneto-telluric profiling would be justified in order to study the deep crustal structure in this sedimentary basin. Interpretational models capable of handling lateral inhomogenities must be developed in order to understand fully the resistivity structure in this and other similar areas.

Before interpretation of upper mantle conductivities is attempted a thorough investigation must be made of source characteristics as they pertain to Alberta. Until this step has been accomplished, further magnetotelluric results for deep lying resistivities must be regarded as ambiguous.

Although data analysis techniques are presently well in advance of interpretational possibilities, some degree of further sophistication is still thought necessary. An extremely useful development would be techniques of pre-whitening in which the filter is individually tailored to each spectra. This would circumvent a major problem in the accurate estimation of spectra not only in magnetotelluric studies but many other disciplines which currently make use of spectral analysis techniques. Similarly, techniques should also be developed to use the polarization properties of magnetotelluric signals to enhance the signal to noise ratio in the same manner as seismic REMODE filters are used.

The spectral bandwidth of the recording instruments must be extended to expand the interpretational possibilities. The technical development of sensors and the conversion of recording systems to digital format will assist in this improvement.

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APPENDICES

Appendix I: Tensor Element Calculations

Section A-I-1: Calculation of Impedance Estimates.

The basic impedance relationship is given by:

$$\vec{E} = Z \vec{H}$$

or:

$$E_X(\omega) = z_{11} H_X(\omega) + z_{12} H_Y(\omega)$$

2.5 - 2

$$E_Y(\omega) = z_{21} H_X(\omega) + z_{22} H_Y(\omega).$$

Post-multiplying each of the above equations successively by $E_X^*(\omega)$, $E_Y^*(\omega)$, $H_X^*(\omega)$, $H_Y^*(\omega)$ (where * denotes the complex conjugate) we have two sets of four equations each:

$$\left\langle \begin{Bmatrix} E_X^* \\ E_Y^* \\ H_X^* \\ H_Y^* \end{Bmatrix} E_X \right\rangle = z_{11} \left\langle \begin{Bmatrix} E_X^* \\ E_Y^* \\ H_X^* \\ H_Y^* \end{Bmatrix} H_X \right\rangle + z_{12} \left\langle \begin{Bmatrix} E_X^* \\ E_Y^* \\ H_X^* \\ H_Y^* \end{Bmatrix} H_Y \right\rangle$$

A-1a, b, c, d

$$\left\langle \begin{Bmatrix} E_X^* \\ E_Y^* \\ H_X^* \\ H_Y^* \end{Bmatrix} E_Y \right\rangle = z_{21} \left\langle \begin{Bmatrix} E_X^* \\ E_Y^* \\ H_X^* \\ H_Y^* \end{Bmatrix} H_X \right\rangle + z_{22} \left\langle \begin{Bmatrix} E_X^* \\ E_Y^* \\ H_X^* \\ H_Y^* \end{Bmatrix} H_Y \right\rangle$$

A-2a, b, c, d

where $\langle \rangle$ denotes a cross or auto power spectral density.

Solving the A-1 set of equations we find solutions are of the form:

From A-1a and A-1b

$$(1)_{z_{11}} = \frac{\langle E_x^* H_Y \rangle \langle E_Y^* E_X \rangle - \langle E_x^* E_X \rangle \langle E_Y^* H_Y \rangle}{\langle E_x^* H_Y \rangle \langle E_Y^* H_X \rangle - \langle H_X^* H_X \rangle \langle E_Y^* H_Y \rangle}$$

$$(1)_{z_{12}} = \frac{\langle E_x^* E_X \rangle \langle E_Y^* H_X \rangle - \langle E_x^* H_X \rangle \langle E_Y^* E_X \rangle}{\langle E_x^* H_Y \rangle \langle E_Y^* H_X \rangle - \langle E_x^* H_X \rangle \langle E_Y^* H_Y \rangle}$$

From A-1a and A-1c

$$(2)_{z_{11}} = \frac{\langle E_x^* H_Y \rangle \langle H_X^* E_X \rangle - \langle E_x^* E_X \rangle \langle H_X^* H_Y \rangle}{\langle E_x^* H_Y \rangle \langle H_X^* H_X \rangle - \langle E_x^* H_X \rangle \langle H_X^* H_Y \rangle}$$

$$(2)_{z_{12}} = \frac{\langle E_x^* E_X \rangle \langle H_X^* H_X \rangle - \langle E_x^* H_X \rangle \langle H_X^* E_X \rangle}{\langle E_x^* H_Y \rangle \langle H_X^* H_X \rangle - \langle E_x^* H_X \rangle \langle H_X^* H_Y \rangle}$$

From A-1a and A-1d

$$(3)_{z_{11}} = \frac{\langle E_x^* H_Y \rangle \langle H_Y^* E_X \rangle - \langle E_x^* E_X \rangle \langle H_Y^* H_Y \rangle}{\langle E_x^* H_Y \rangle \langle H_Y^* H_X \rangle - \langle E_x^* H_X \rangle \langle H_Y^* H_Y \rangle}$$

$$(3)_{z_{12}} = \frac{\langle E_x^* E_X \rangle \langle H_Y^* H_X \rangle - \langle E_x^* H_X \rangle \langle H_Y^* E_X \rangle}{\langle E_x^* H_Y \rangle \langle H_Y^* H_X \rangle - \langle E_x^* H_X \rangle \langle H_Y^* H_Y \rangle}$$

From A-1b and A-1c

$$(4)_{z_{11}} = \frac{\langle E_Y^* H_Y \rangle \langle H_X^* E_X \rangle - \langle E_Y^* E_X \rangle \langle H_X^* H_Y \rangle}{\langle E_Y^* H_Y \rangle \langle H_X^* H_X \rangle - \langle E_Y^* H_X \rangle \langle H_X^* H_Y \rangle}$$

$$(4)_{z_{12}} = \frac{\langle E_Y^* E_X \rangle \langle H_X^* H_X \rangle - \langle E_Y^* H_X \rangle \langle H_X^* E_X \rangle}{\langle E_Y^* H_Y \rangle \langle H_X^* H_X \rangle - \langle E_Y^* H_X \rangle \langle H_X^* H_Y \rangle}$$

From A-1b and A-1d

$$(5)_{z_{11}} = \frac{\langle E_Y^* H_Y \rangle \langle H_Y^* E_X \rangle - \langle E_Y^* E_X \rangle \langle H_Y^* H_Y \rangle}{\langle E_Y^* H_Y \rangle \langle H_Y^* H_X \rangle - \langle E_Y^* H_X \rangle \langle H_Y^* H_Y \rangle}$$

$$(5)_{z_{12}} = \frac{\langle E_Y^* E_X \rangle \langle H_Y^* H_X \rangle - \langle E_Y^* H_X \rangle \langle H_Y^* E_X \rangle}{\langle E_Y^* H_Y \rangle \langle H_Y^* H_X \rangle - \langle E_Y^* H_X \rangle \langle H_Y^* H_Y \rangle}$$

Finally, from A-1c and A-1d

$$(6)_{z_{11}} = \frac{\langle H_X^* H_Y \rangle \langle H_Y^* E_X \rangle - \langle H_X^* E_X \rangle \langle H_Y^* H_Y \rangle}{\langle H_X^* H_Y \rangle \langle H_Y^* H_X \rangle - \langle H_X^* H_X \rangle \langle H_Y^* H_Y \rangle}$$

$$(6)_z = \frac{\langle H_X^* E_X \rangle \langle H_Y^* H_X \rangle - \langle H_X^* H_X \rangle \langle H_Y^* E_X \rangle}{\langle H_X^* H_Y \rangle \langle H_Y^* H_X \rangle - \langle H_X^* H_X \rangle \langle H_Y^* H_Y \rangle}$$

Similarly, solutions can be obtained for z_{21} and z_{22} .

Section A-I-2: Polarized and Unpolarized Impedance Estimates.

The effect of an unpolarized magnetotelluric field on the impedance estimates calculated in Section A-I-1 will now be considered.

If the magnetotelluric fields are unpolarized then $\langle H_x^* H_y \rangle$, $\langle E_x^* E_y \rangle$, $\langle E_x^* H_x \rangle$, and $\langle E_y^* H_y \rangle$ should tend to zero. The z_{11} and z_{12} impedance estimates will then have the form:

$$(1) z_{11} \approx \frac{E_x}{H_x}$$

$$(1) z_{12} \approx \frac{\langle E_x^* E_x \rangle}{\langle E_x^* H_y \rangle} = \frac{E_x}{H_y}$$

$$(2) z_{11} \approx \frac{E_x}{H_x}$$

$$(2) z_{12} \approx \frac{\langle E_x^* E_x \rangle}{\langle E_x^* H_y \rangle} = \frac{E_x}{H_y}$$

$$(3) z_{11} - \text{indeterminate}$$

$$(3) z_{12} - \text{indeterminate}$$

$$(4) z_{11} - \text{indeterminate}$$

$$(4) z_{12} - \text{indeterminate}$$

$$(5) z_{11} \approx \frac{E_x}{H_x}$$

$$(5) z_{12} \approx \frac{\langle H_Y^* E_X \rangle}{\langle H_Y^* H_Y \rangle} = \frac{E_X}{H_Y}$$

$$(6) z_{11} \approx \frac{E_X}{H_X}$$

$$(6) z_{12} \approx \frac{\langle H_Y^* E_X \rangle}{\langle H_Y^* H_Y \rangle} = \frac{E_X}{H_Y}$$

and similar expressions exist for z_{21} and z_{22} . Thus two of the estimates are unstable ($^{(3)}z$, $^{(4)}z$) and the four remaining estimates are stable and predict

$$z_{ij} = \frac{E_i}{H_j}.$$

Section A-I-3: The Effect of Noise on the Tensor Elements estimates.

The effect of incoherent recorded noise on the stable unpolarized impedance element estimates can be seen as follows:

$$\begin{aligned}
 \text{Let: } E &= E^{\text{SIGNAL}} + E^{\text{NOISE}} = E^S + E^n \\
 H &= H^{\text{SIGNAL}} + H^{\text{NOISE}} = H^S + H^n \\
 \langle E^* E \rangle &= \langle (E^S + E^n)^* (E^S + E^n) \rangle \\
 &= \langle E^{S*} E^S \rangle + \langle E^{S*} E^n \rangle + \langle E^{n*} E^S \rangle + \langle E^{n*} E^n \rangle
 \end{aligned}$$

and if the noise is truly incoherent,

$$\langle E^* E \rangle = \langle E^{S*} E^S \rangle + \langle E^{n*} E^n \rangle.$$

Similarly

$$\langle H^* H \rangle = \langle H^{S*} H^S \rangle + \langle H^{n*} H^n \rangle.$$

Also

$$\begin{aligned}
 \langle E^* H \rangle &= \langle (E^S + E^n)^* (H^S + H^n) \rangle \\
 &= \langle E^{S*} H^S \rangle + \langle E^{S*} H^n \rangle + \langle E^{n*} H^S \rangle + \langle E^{n*} H^n \rangle \\
 &= \langle E^{S*} H^S \rangle
 \end{aligned}$$

Therefore

$${}_{1,2}Z_{12}, \quad \frac{\langle E_X^* E_X \rangle}{\langle E_X^* H_Y \rangle} = \frac{\langle E_X^{S*} E_X^S \rangle + \langle E_X^{n*} E_X^n \rangle}{\langle E_X^{S*} H_Y^S \rangle}$$

and

$${}_{5,6}Z_{12}, \quad \frac{\langle H_Y^* E_X \rangle}{\langle H_Y^* H_Y \rangle} = \frac{\langle H_Y^{S*} E_X^S \rangle}{\langle H_Y^{S*} H_Y^S \rangle + \langle H_Y^{n*} H_Y^n \rangle}$$

since

$z_{12} = \frac{E_x}{H_y}$ represents the true impedance element in an

unpolarized field, then:

$${}^{1,2}z_{12}' = z_{12} \left\{ 1 + \frac{\langle E_x^{n*} E_x^n \rangle}{\langle E_x^{s*} E_x^s \rangle} \right\}$$

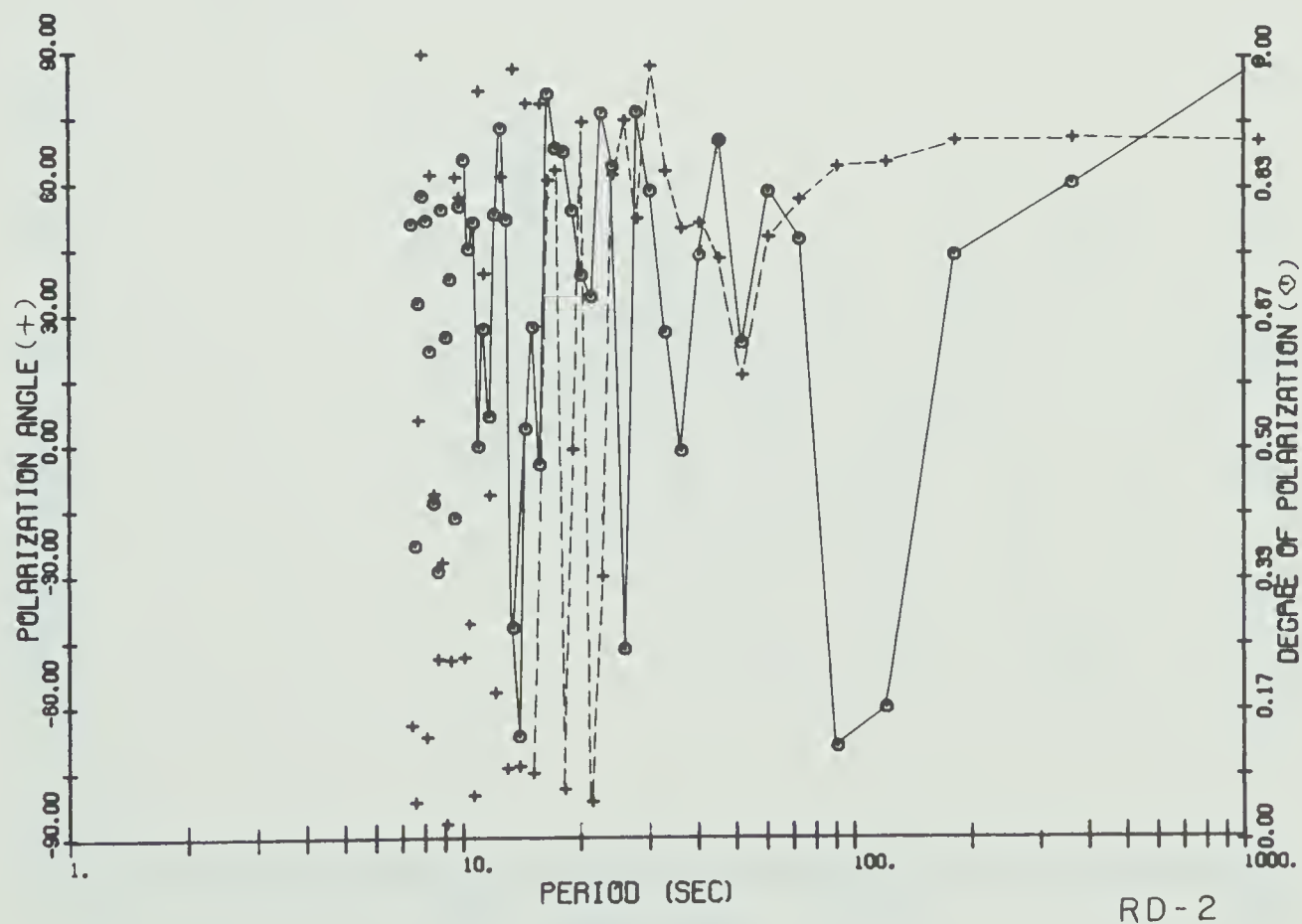
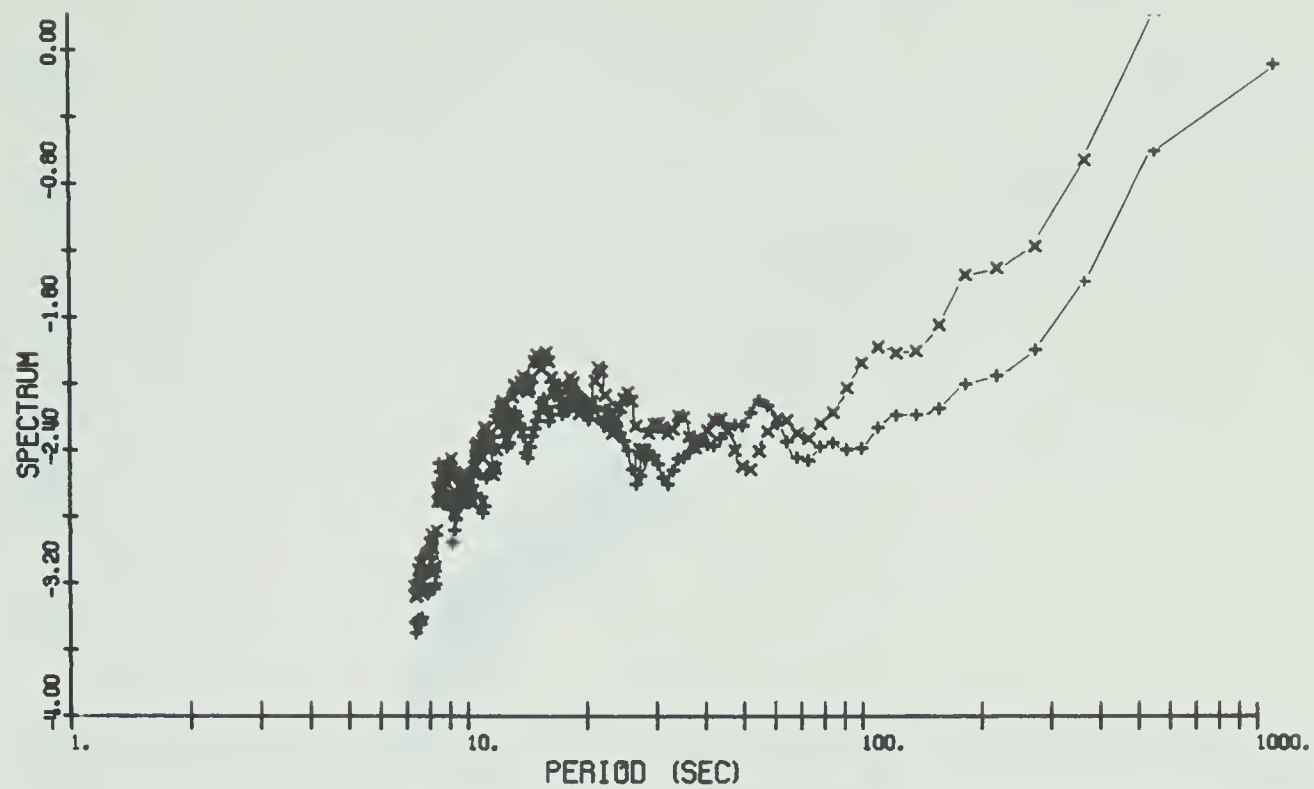
and

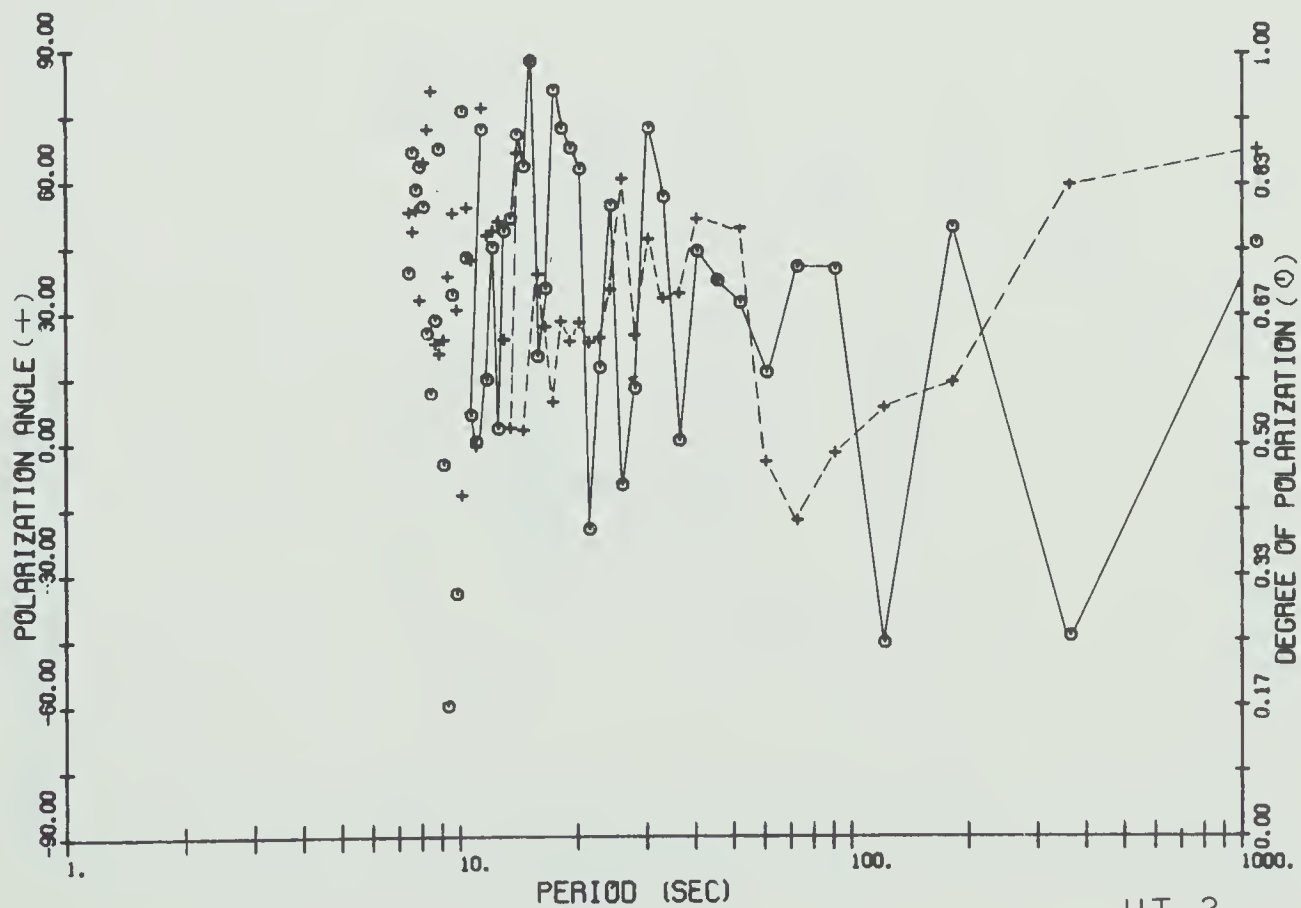
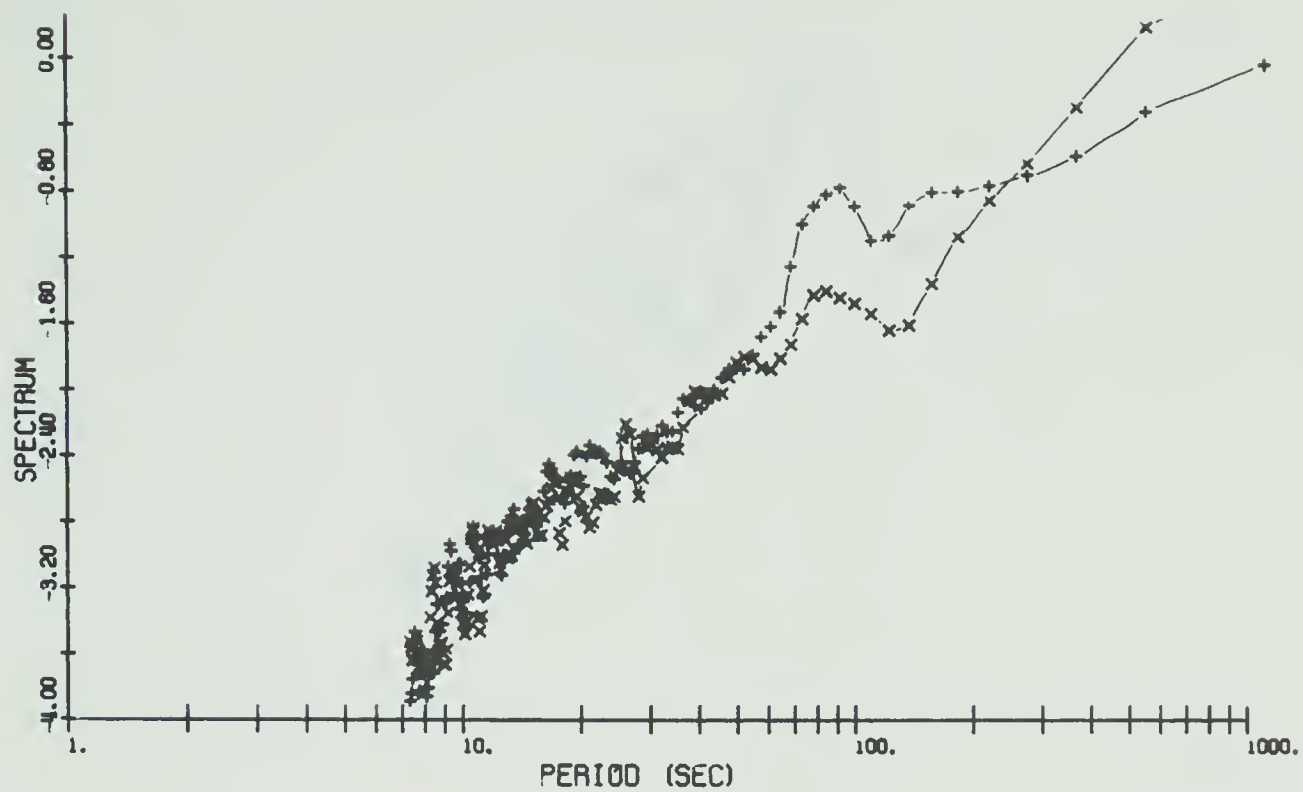
$${}^{5,6}z_{12}' = z_{12} / \left\{ 1 + \frac{\langle H_y^{n*} H_y^n \rangle}{\langle H_y^{s*} H_y^s \rangle} \right\}$$

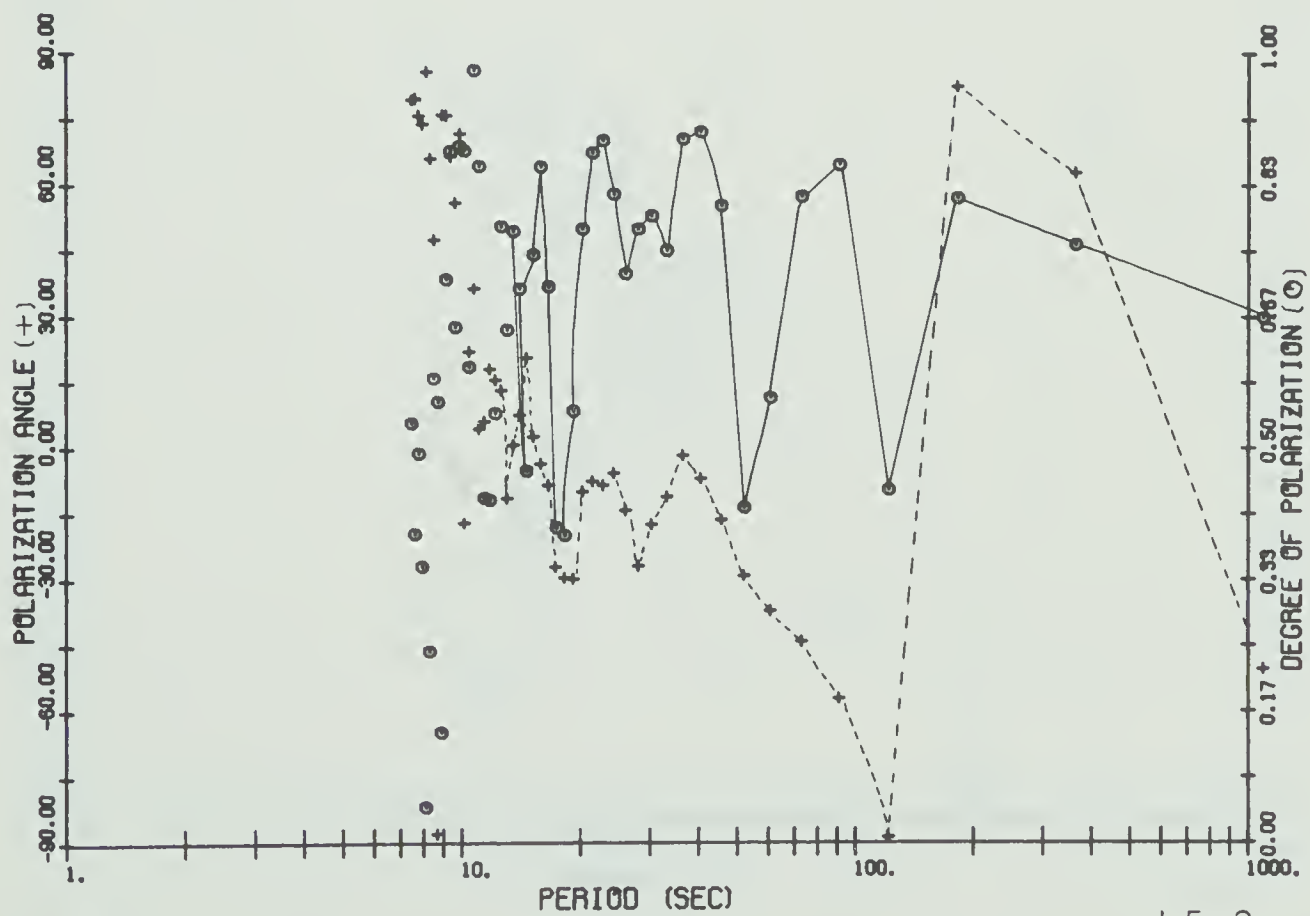
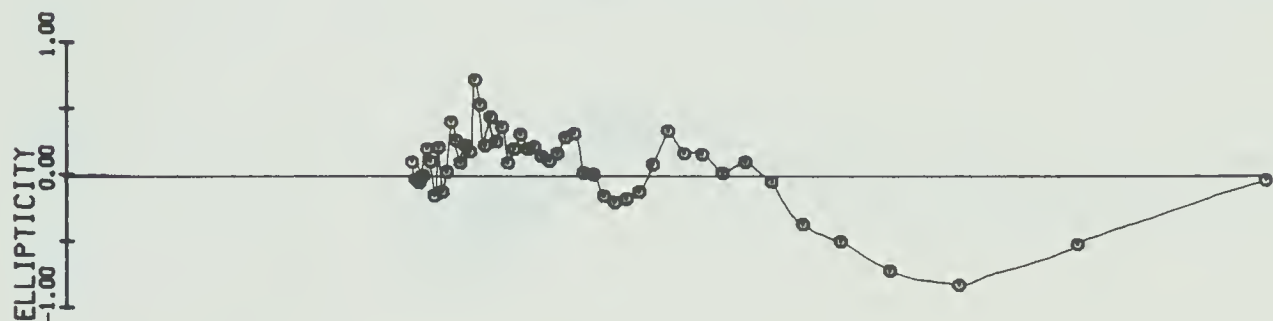
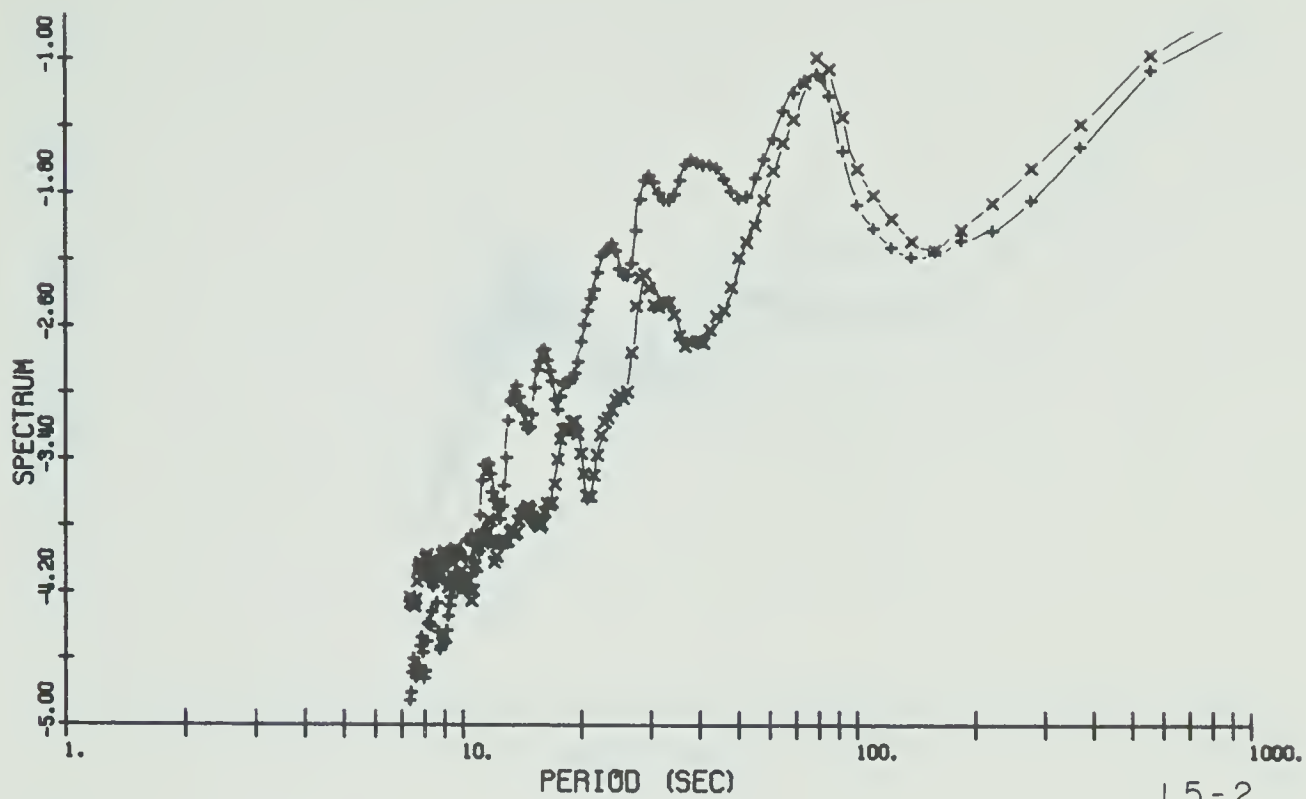
Therefore it can be seen that if the magnetotelluric fields are unpolarized, then ${}^{1,2}z_{12}'$ gives a biased estimate greater than the true value of the impedance element z_{12} due to recorded electric noise while ${}^{5,6}z_{12}'$ gives a low biased estimate of z_{12} due to recorded magnetic noise. The z_{21} estimates are effected in a similar manner.

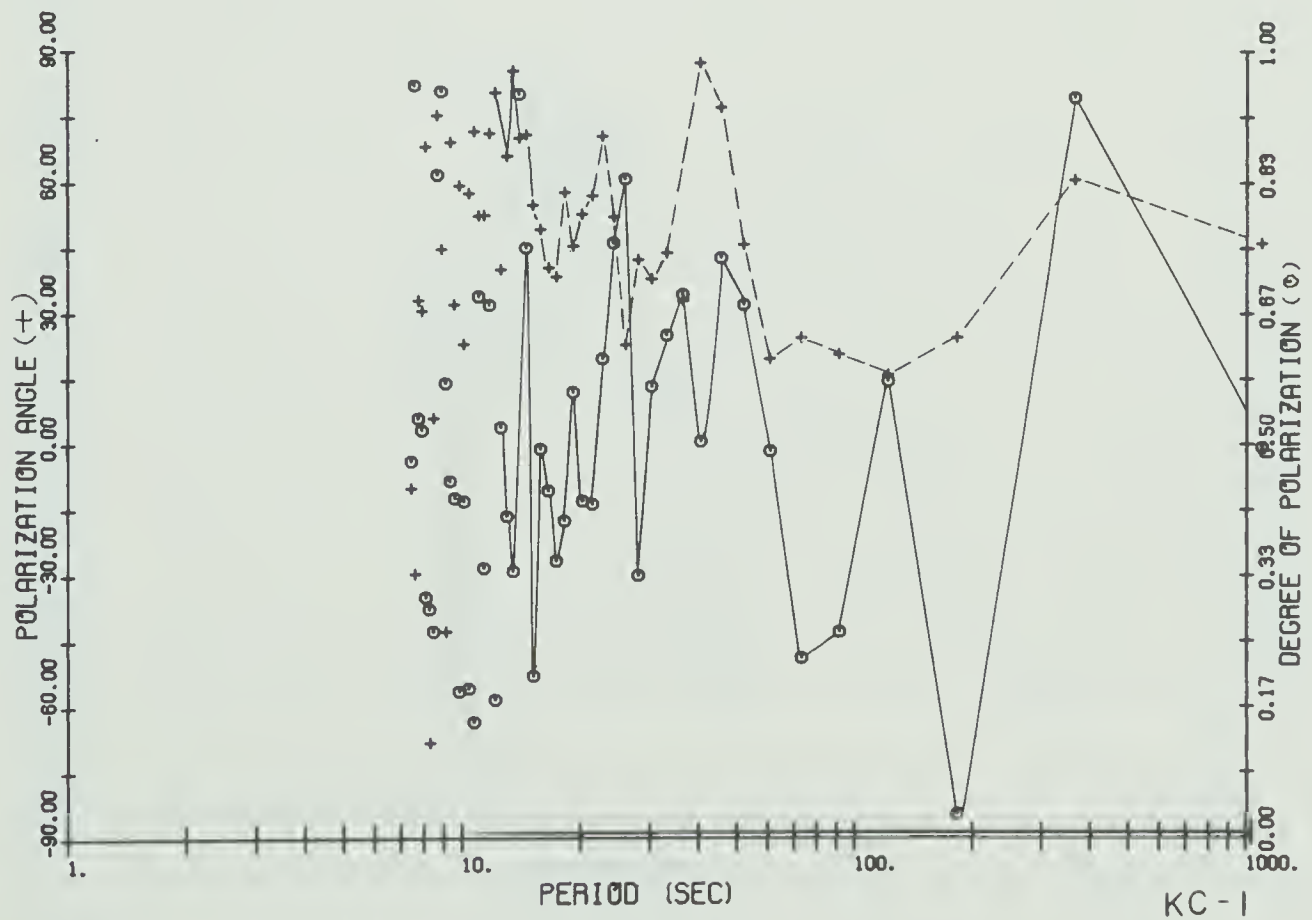
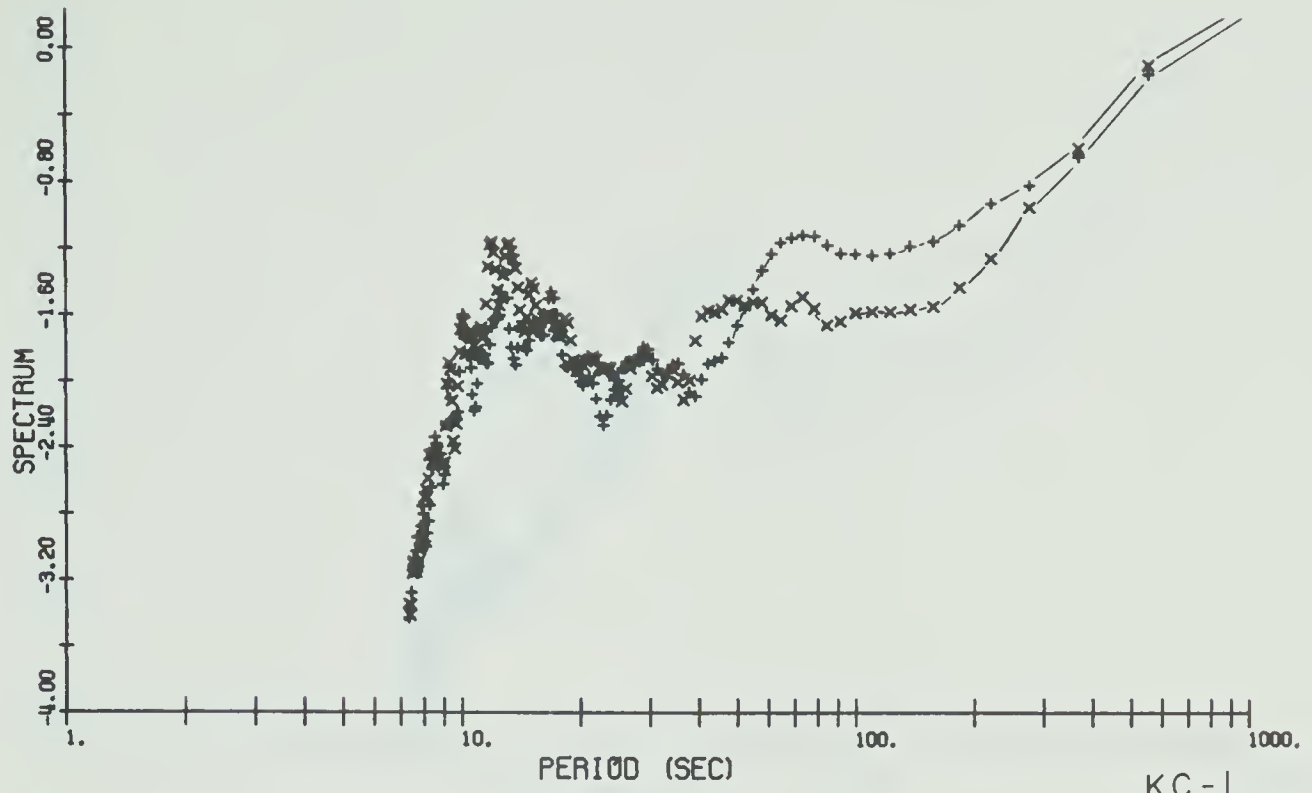
This appendix shows the general form of the magnetic power spectra and the magnetic polarization characteristics for data collected in this survey. The power spectra for Hx and Hy (non-rotated) as presented are instrument pre-whitened spectra (see Chapter 3) and thus are not the actual power spectra in γ^2/Hz . The Hx spectra is denoted by + and the Hy spectra denoted by x.

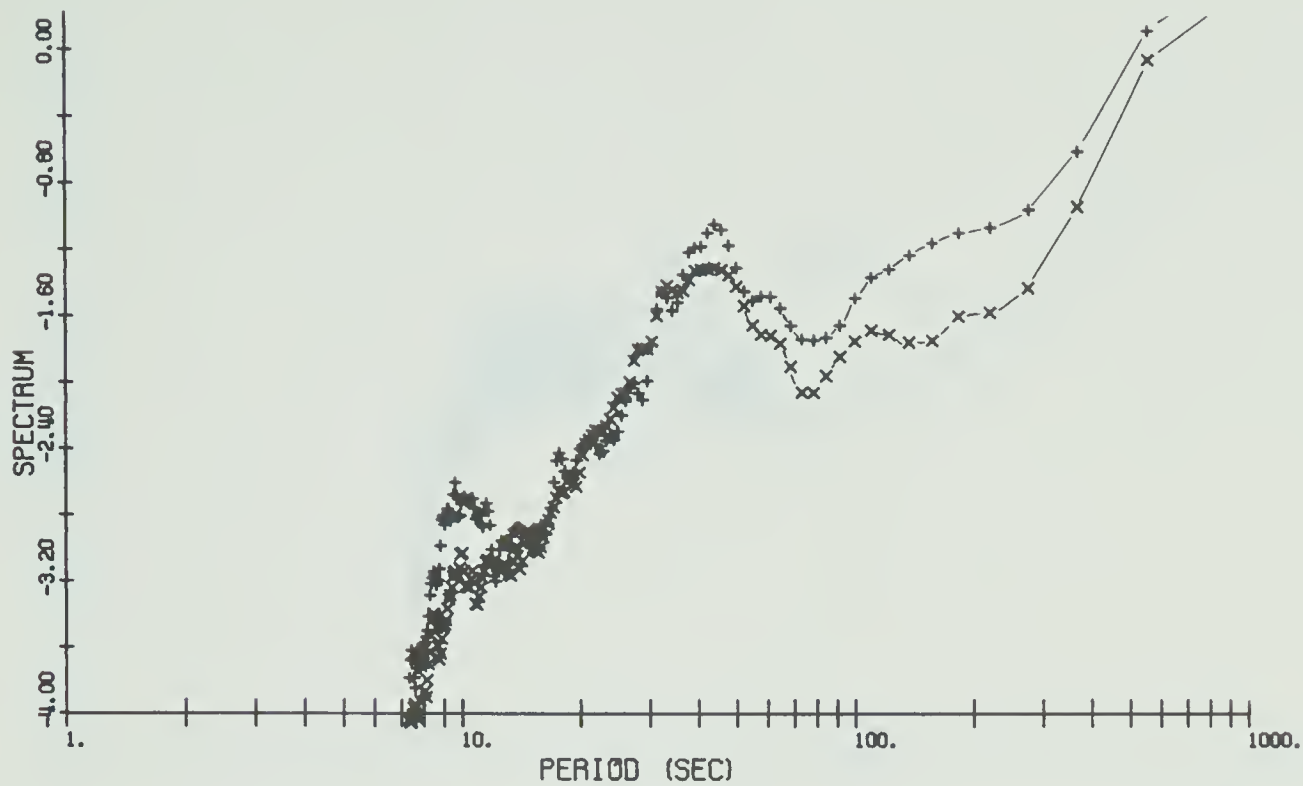
The polarization characteristics of the geomagnetic micropulsations were digitally determined using a quasi-monochromatic wave train theory of physical optics combined with spectral analysis as outlined by Fowler et al (1967). The angle of polarization and the ellipticity are thus calculated for the polarized portion of the signal. The polarization angle is shown as a dashed line and the degree of polarization is a solid line.



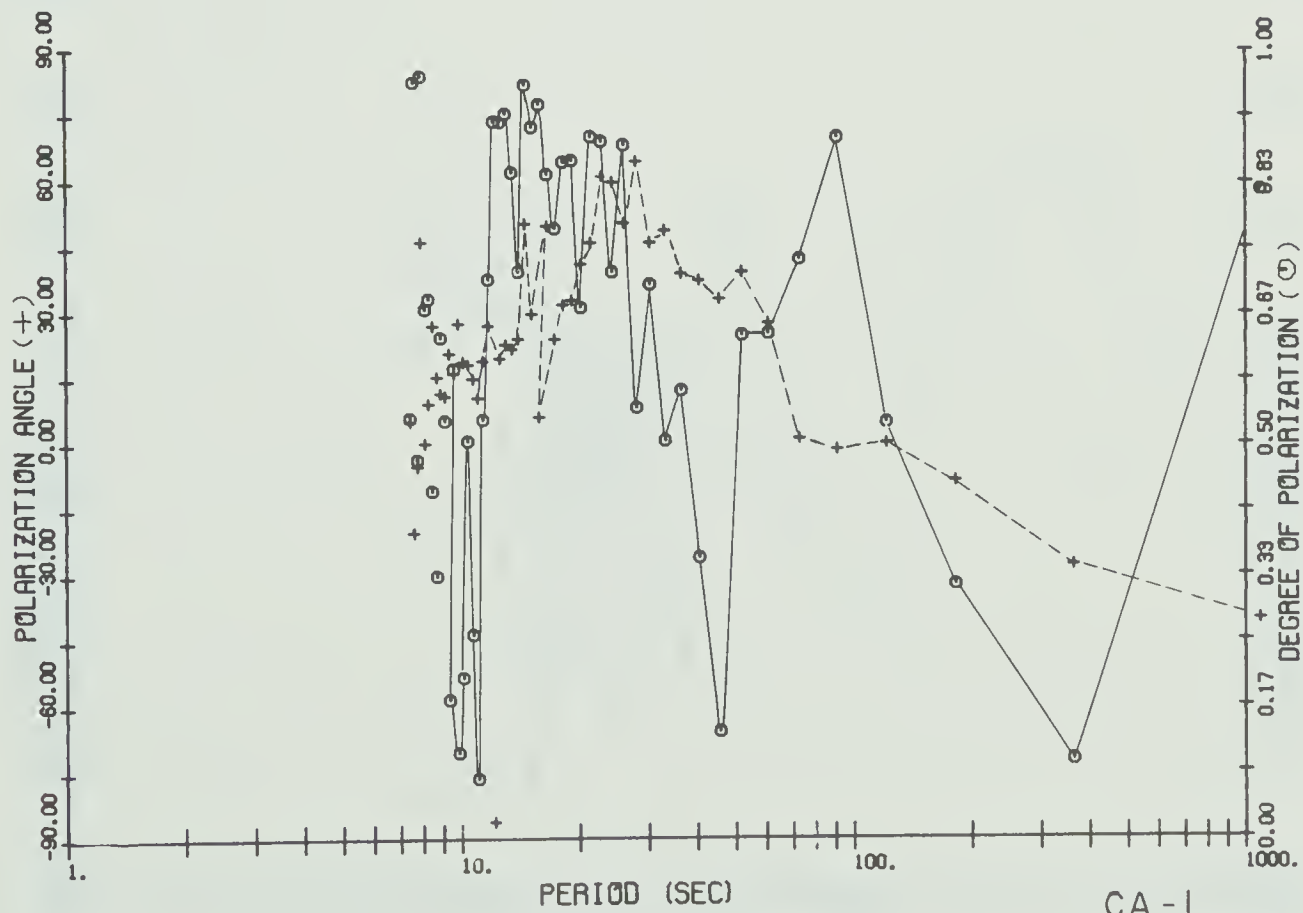
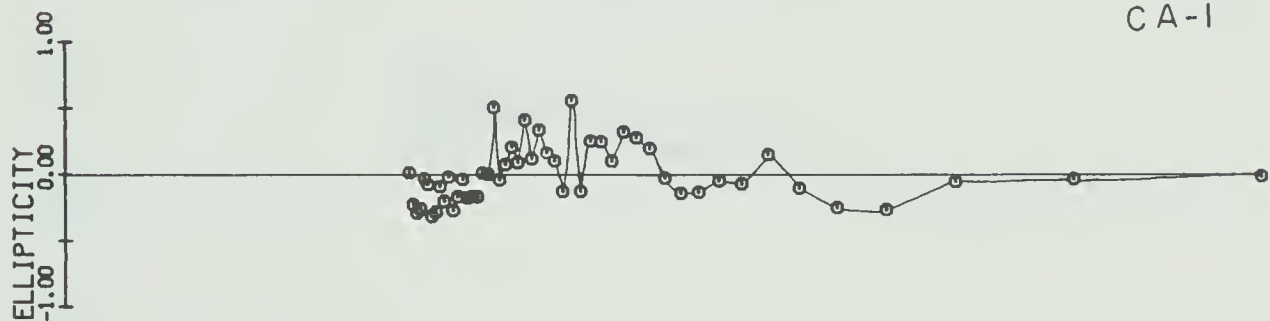




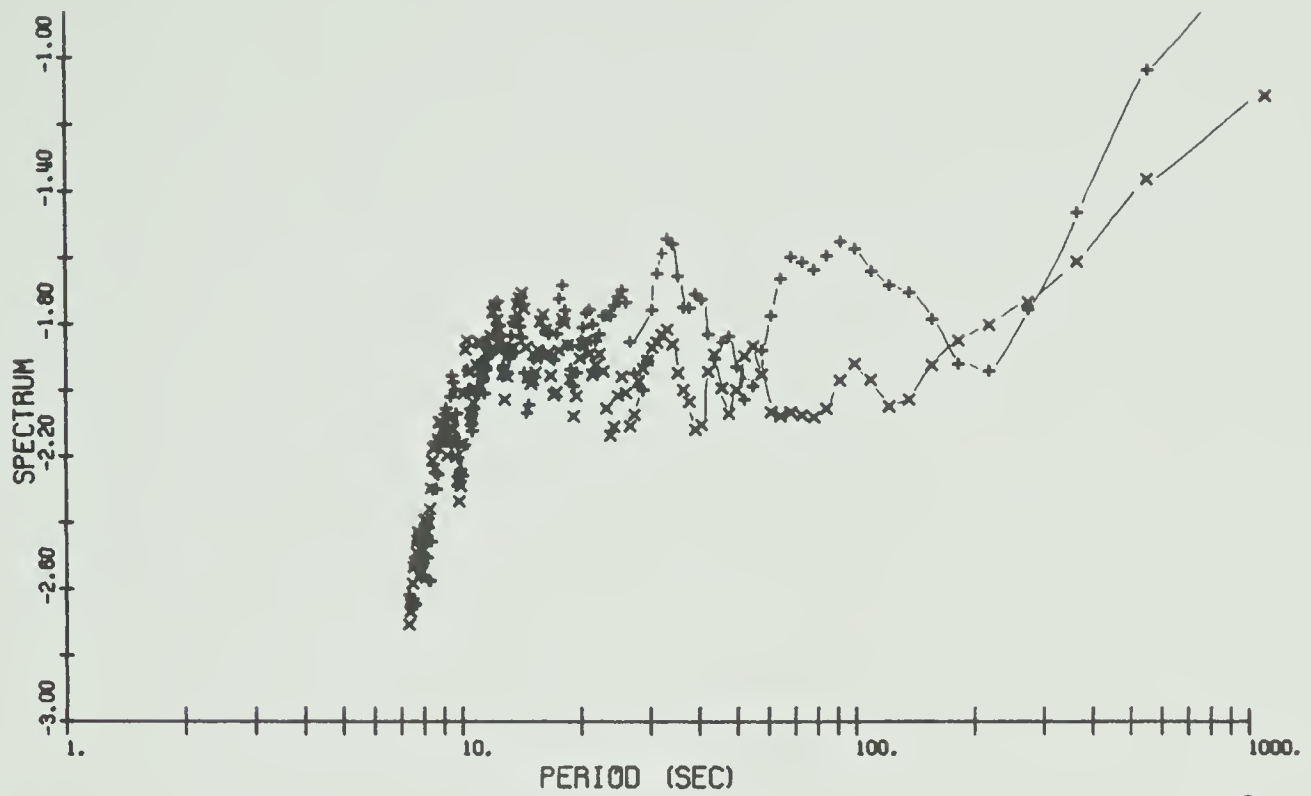




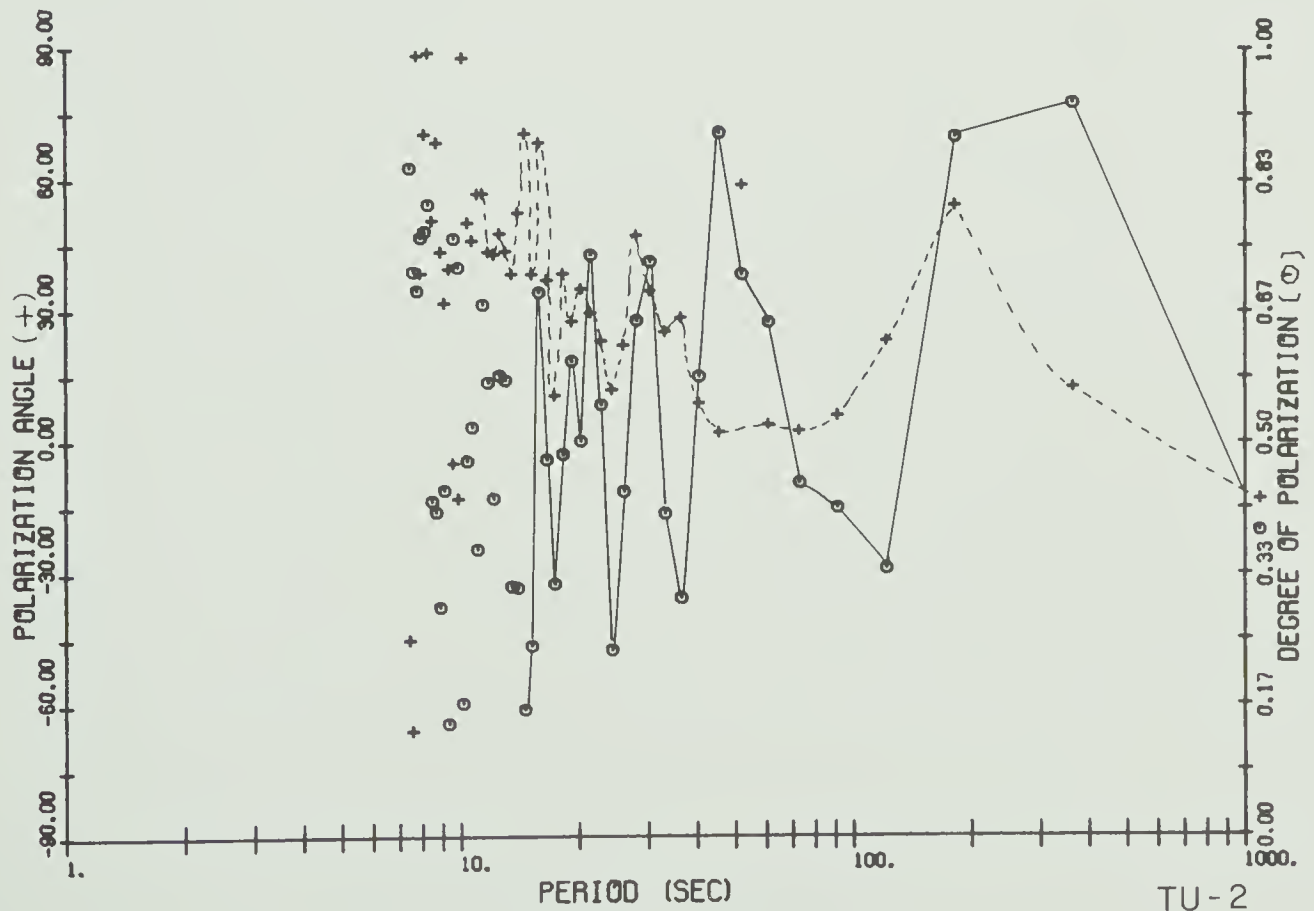
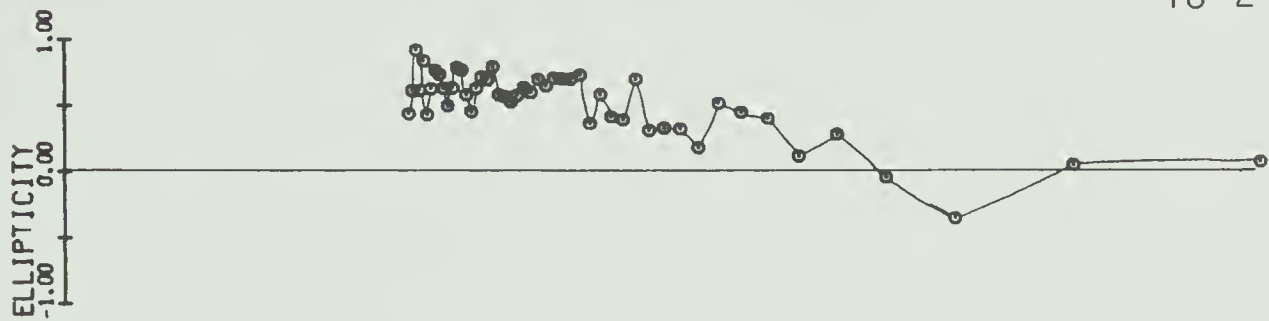
CA-1



CA-1



TU-2



TU-2

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